MODULE 3A

Syllabus: **DC Machines:** (a) Principle of operation, constructional details, induced emf expression, types of generators, and the relation between induced emf and terminal voltage. (b) Principle of operation, back emf and torque equations, types of motors, characteristics (shunt and series only), and applications.

Single Phase Transformer: Necessity of transformer, the principle of operation, Types, and construction of single-phase transformers, emf equation, losses, variation of losses with respect to load, efficiency, and condition for maximum efficiency.

Introduction:

- An electrical machine, deals with energy transfer either from mechanical to electrical or electrical to mechanical is called **DC Machine.**
- The DC machines are classified into
- **i) DC Generator**
- **ii) DC Motor**
- **DC Generator:** The machine which converts mechanical energy into Electrical energy
- **DC motor:** The machine which converts Electrical energy into Mechanical energy

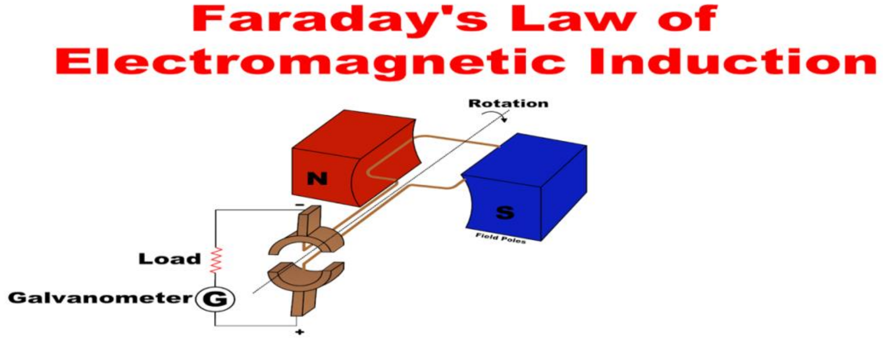
Working principle of D.C.Machine as a generator

Working principle of D.C.Machine as a generator:

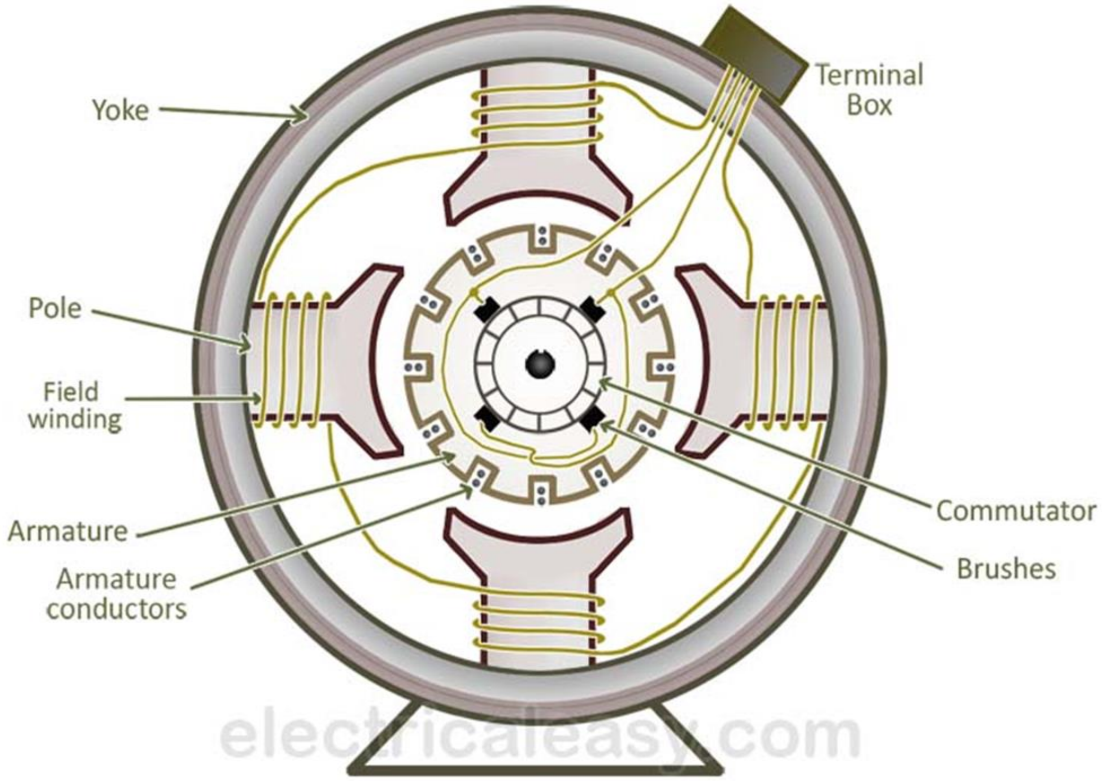
- It is based on the principle of **dynamically induced e.m.f .**
- Whenever a conductor cuts magnetic flux, dynamically induced e.m.f. is produced in the conductor according to the Faradays laws of Electromagnetic Induction. This e.m.f. causes a current to flow in the circuit, if the conductor circuit is closed.
- The emf is given by

  $$e = B * l * v * \sin\theta$$ volts/coil side where,

  Where B=the flux density in Tesla,
  l=the active length of the coil side in meters
  v=the velocity with which the coil is moved in meters/sec and
  θ is the angle between the direction of the flux and relative velocity.
- The direction of the induced voltage can be obtained by applying **Fleming's right hand rule.**



Construction of DC Machine



Salient parts of a D.C.Machine are:

(i)Yoke
ii) Field system (poles)
(iii) Armature
(iv) Commutator
(v)Brushes

Yoke:

It is made of **cast iron or silicon steel**

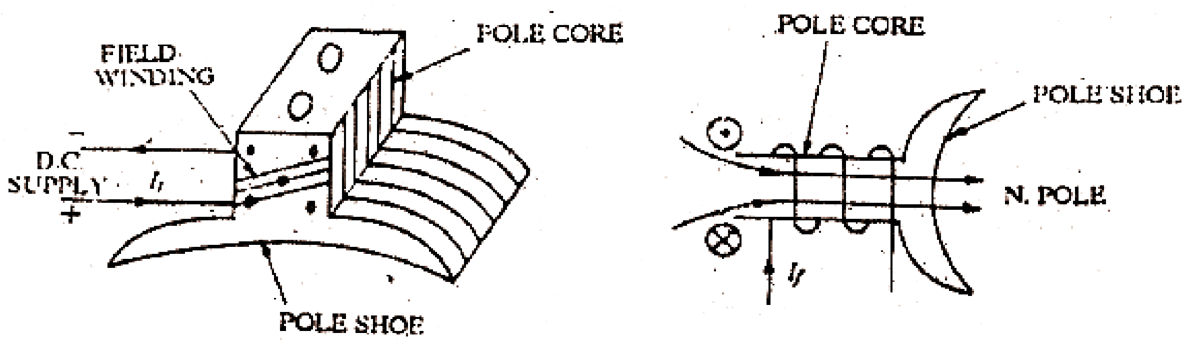
- It serves the purpose of **outermost cover** of the D.C. machine. So that the insulating materials get protected from harmful atmospheric elements like moisture, dust and various gases like SO2, acidic fumes etc.
- It provides mechanical support to the poles, It forms a part of the magnetic circuit and it provides a path of low reluctance for magnetic flux.

Poles:

It is made cast iron or cast steel laminations which are stamped together.

Each pole is divided into two parts a) **pole core** and b) **pole shoe**

  - Pole core basically carries a field winding which is necessary to produce the flux.
  - It directs the flux produced through air gap to armature core and to the next pole.
- Pole shoe enlarges the area of armature core to come across the flux, which is necessary to produce larger induced emf. To achieve this, pole shoe has given a particular shape



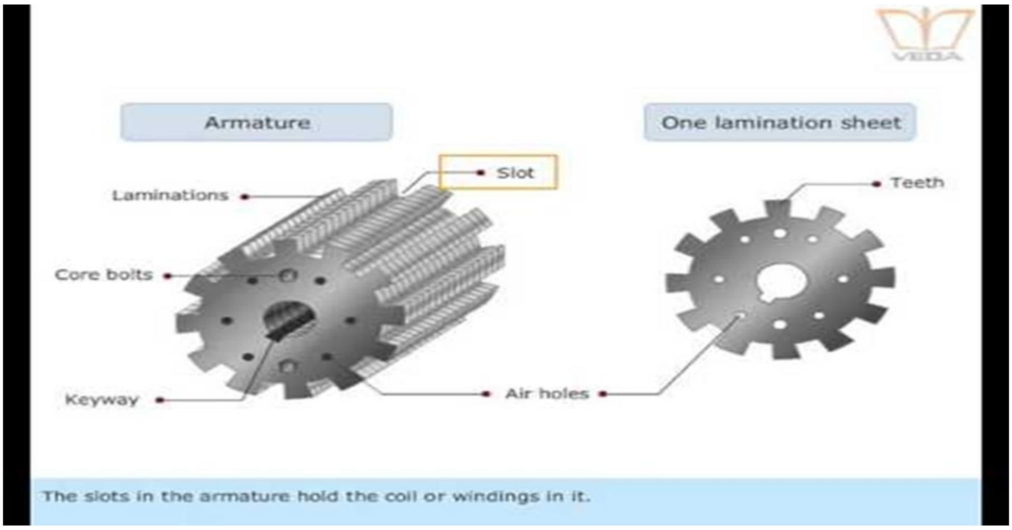
Field winding [F1-F2]:

It is made of conducting material like copper or Aluminum. The field winding is wound on the pole core with a definite direction.

- It carries current due to which the pole core behaves as an electromagnet and produces necessary flux. As it's exciting the pole as electromagnet hence it is also called Exciting winding.

Armature:

- It is further divided into two parts namely,
  - Armature core and
  - Armature winding
- Armature core is cylindrical in shape made up of iron and mounted on the shaft. It is provided with of slots on its outer periphery to place the conductor and the air ducts to permit the air flow through armature which serves cooling purpose.
- In order to collect the Emf generated in each conductor they are connected in certain pattern called armature winding.



Commutator:

- The basic nature of Emf induced in the armature conductors is alternating. This needs rectifications in case of D.C. generator which is possible by device called commutator.
- It is cylindrical in shape made of hard drawn copper segments. These segments are insulated from each other by a layer of mica.

Brushes and brush gear:

- Brushes collect current from commutator and make it available to the stationary external circuit.
- Ball bearings are usually used as they are more reliable.
- For heavy duty machines, roller bearings are preferred.

Types of D.C. Armature Windings

Lap Winding	Wave Winding
In this winding all the pole groups of the coils generating emf in the same direction at any instant of time are connected in parallel by the brushes.	In this winding all the coils carrying current in the same direction are connected in series and coils carrying current in opposite direction are connected in other series circuit.
2. Lap winding is also known as parallel windings.	2. Wave winding is also known as series winding.
3. The number of parallel path is equal to the number of poles i.e., A = P.	3. The number of parallel paths is always equal to 2 i.e., A = 2.
4. The number of brush required by this winding is always equal to the number of poles.	4. The number of brushes required by this winding is always equal to 2.
5. Lap windings are used for low voltage and high current machines.	6. Wave windings are used for high voltage and low current machines.

Emf Equation of DC Generator:

Let,

Φ = Flux produced by each pole in weber (Wb) and

P = number of poles in the DC generator.

N = speed of the armature conductor in rpm.

Consider a one revolution of the conductor

Total flux produced by all the poles $= \emptyset \times \mathbf{P}$

Time taken to complete one revolution $= \frac{\mathbf{60}}{\mathbf{N}}$

Now, according to Faraday's law of induction, the induced EMF of the conductor is equal to rate of change of flux.

$$e = \frac{d\phi}{dt} \quad and \; e = \frac{total\ flux}{time\ take}$$

Therefore,

Induced EMF of one conductor is

$$e = \frac{\phi P}{\frac{60}{N}} = \phi P \frac{N}{60}$$

Let us suppose there are **Z** total numbers of conductor in a generator, and arranged in such a manner that all parallel paths are always in series.
Here, Z = total numbers of conductor A = number of parallel paths
Then, **Z/A = number of conductors connected in series**

Therefore,
Induced EMF of DC generator
E_g = EMF of one conductor × number of conductor connected in series.

Induced Emf of DC generator is

$$e = \phi P \frac{N}{60} X \frac{Z}{A} \; volts$$

$$\mathbf{e} = \frac{\emptyset PNZ}{60A}$$

Problems on Emf equation

Formula

$$E_g = \frac{\phi Z N}{60} X \frac{P}{A}\, volt$$

Φ- flux produced by each pole in wb
Z- total no of conductors in armature
N- speed armature in rpm
P- No of poles
A- No of parallel paths , for wave winding A=2
for lap winding A=P

1. **A 4 pole, 1500 rpm DC generator has a lap wound armature having 24 slots with 10 conductors per slot. If the flux per pole is 0.04 Wb, calculate EMF generated in the armature. What would be the generated EMF if the winding is wave connected?**

Solution:

Given: P = 4, N = 1500rpm, Lap i. e. **A = P = 4**, $\emptyset = 0.04$ Wb

Z = Slots X Conductors per Slot = 24 X 10 = 240

$$E_g = \frac{\emptyset PNZ}{60A} = \frac{0.04 \times 4 \times 1500 \times 240}{60 \times 4} = \mathbf{240\ V}$$

If winding is wave connected, A = 2

$$E_g = \frac{0.04 \times 4 \times 1500 \times 240}{60 \times 2} = \mathbf{480V}$$

2. **A 4 pole generator with wave wound armature has 51 slots each having 24 conductors. The flux per pole is 0.01 weber. At what speed must the armature rotate to give an induced EMF of 220 V? What will be the voltage developed if the winding is lap connected and the armature rotates at the same speed?**

Solution:

Given: P = 4, wave connected hence **A = 2,** 51 slots, 24 conductors per slot, $\emptyset = 0.01$ Wb , $E_g = 220V$

$$E_g = \frac{\emptyset PNZ}{60A}$$ Where **Z = 51 X 24 = 1224**

$$220 = \frac{0.01 \times 4 \times N \times 1224}{60 \times 2}$$

$$N = \frac{220 \times 60 \times 2}{0.01\ X\ 4\ X 1224}$$

i.e **N = 539.2156 r.p.m … speed for 220V**

For lap wound, A = P = 4 and N = 539.2156 r.p.m

$$E_g = \frac{\emptyset PNZ}{60A} = \frac{0.01 \times 4 \times 539.2156 \times 1224}{60 \times 4} = 110V$$

3. **A 8 pole DC generator has 500 armature conductors and useful flux per pole of 0.065 wb. What will be EMF generated if the winding is lap connected and runs at 1000 rpm? What must be the speed at which it is to be driven to produce the same EMF if the winding is Wave connected?**

Solution:

Given: **P = 8 Z= 500 conductors** $\emptyset = \mathbf{0.065\ Wb}$, **N = 1000rpm**

When it is lap connected A=P =8 and E_g =?

$$E_g = \frac{\emptyset PNZ}{60A} = \frac{0.065\ x\ 8\ x1000\ x\ 500}{60\ \ x\ 8} = \mathbf{541.667\ V}$$

ii) N=? When $\mathbf{E_g}$ **= 541.667V** and winding is w**ave** connected I,e **A=2**

$$541.667 = \frac{0.065 \times 8 \times N \times 500}{60 \times 2}$$

$$N = \frac{541.667 \times 60 \times 2}{0.065\ X\ 8\ X\ 500}$$

N = 250 rpm

Types of DC Generators

- Thw field winding is also called as exciting winding. Supplying current to the field winding is excitation.
- Depending upon the method of excitation used in the generators are classified into
  - i) Separately excited DC generator
  - ii) Self-excited DC generator.

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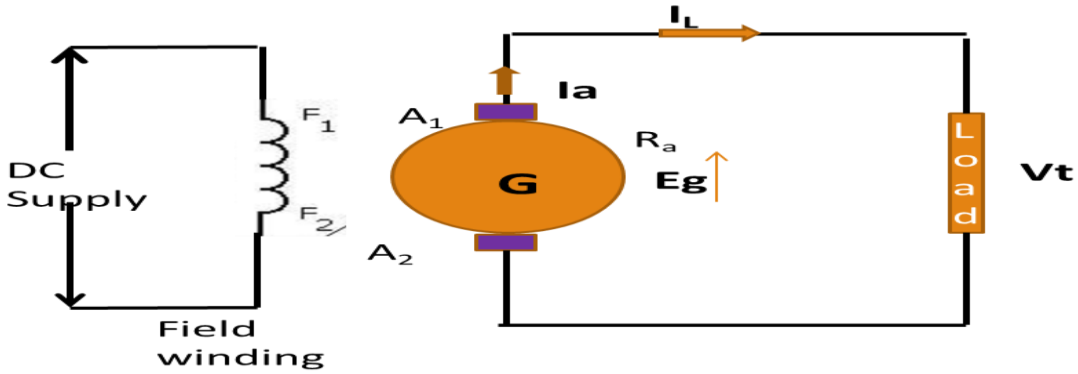
Separately Excited Generators:

In separately excited dc machines, the field winding is supplied from a **separate power source** as shown in below fig.



Eg- generated Emf in generator
Ia – Armature current
R_a - armature resistance
I_L - Load current
Vt- Terminal voltage
F1 and F2 – Terminals of field winding

Self-Excited Field Generators:

- The self-excited DC generator produces a magnetic field by itself without DC sources from an external. The electromotive force that produced by generator at armature winding is supply to a field winding instead of DC source from outside of the generator. Therefore, field winding is necessary connected to the armature winding.
- When generator started, due to residual flux, it develops a small amount of EMF which drives a small current in the field winding. This tends to increase the flux in the poles in turn increases the EMF.This cumulative process continues until generator produces a rated voltage.

They further classified into:

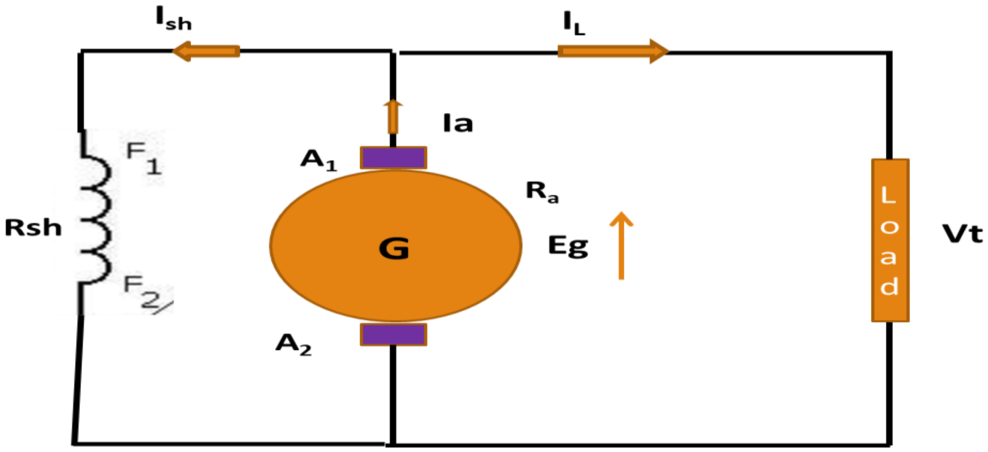
a) DC Shunt generator

b) DC Series generator

c) DC Compound generator.

a) **Shunt generator:**

- In shunt generator, the field winding is connected in **parallel** with the armature winding and combination across the load. As shown in the fig.



Ish- current through shunt field winding   Rsh- Resistance of shunt field winding

From the fig

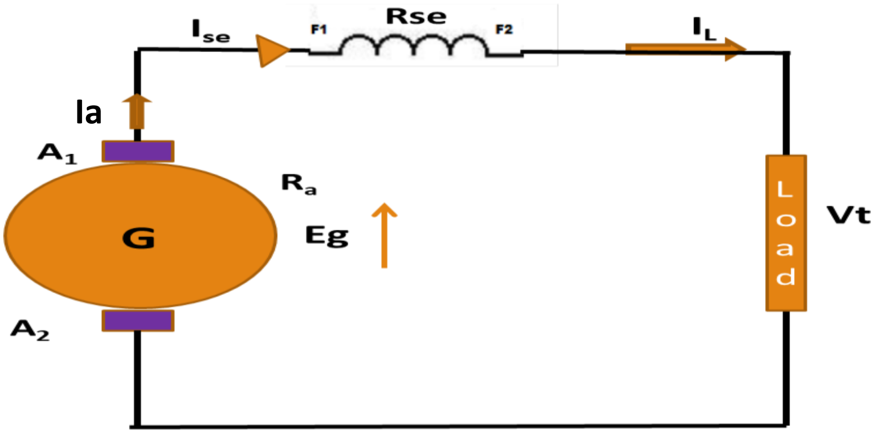
Armature current $I_a = I_L + I_{sh}$ and $I_{sh} = \frac{Vt}{Rsh}$

Induced EMF $E_g = V_t + I_aR_a + V_{brush}$

Terminal voltage $V_t = E_g - I_aR_a - V_{brush}$

b) **Series generator:**

- In series generator, the field winding is connected in **series** with the armature winding and to the load. As shown in the fig.



Ise- current through series field winding   Rse- Resistance of series field winding

From the fig

Armature current $I_a = I_{se} = I_L$

Induced EMF $E_g = V_t + I_aR_a + I_{se}R_{se} + V_{brush}$

$E_g = V_t + Ia(Ra + R_{se}) + V_{brush}$ $[I_a = I_{se}]$

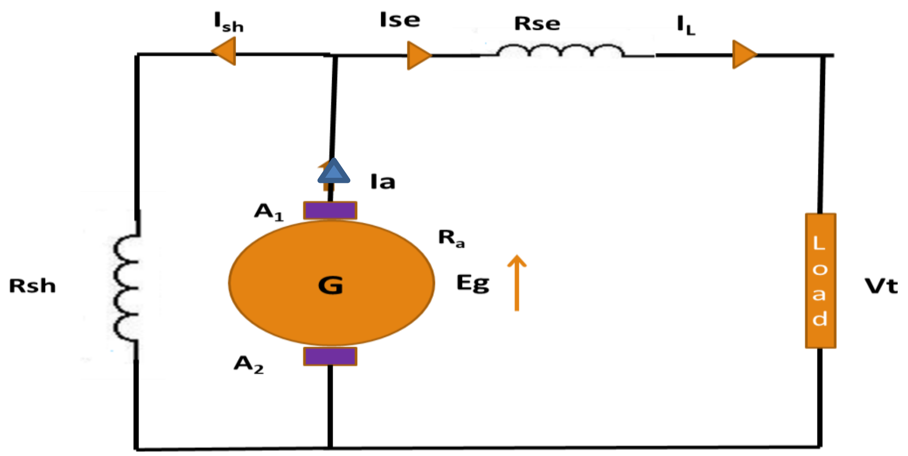
Terminal voltage $V_t = E_g - Ia(Ra + R_{se}) - V_{brush}$

Compound Generator:

- The compound generator has provided with magnetic field in combine with excitation of shunt and series field winding. The part of field winding is connected in parallel with armature called shunt field winding and part in series with armature winding called series field winding.
- There are two types of Compound generators such as
  (i) Long shunt Compound Generator
  (ii) Short Shunt Compound Generator

Short Shunt Compound Generator:

The shunt field winding is connected in parallel only with the armature. As shown in the fig.



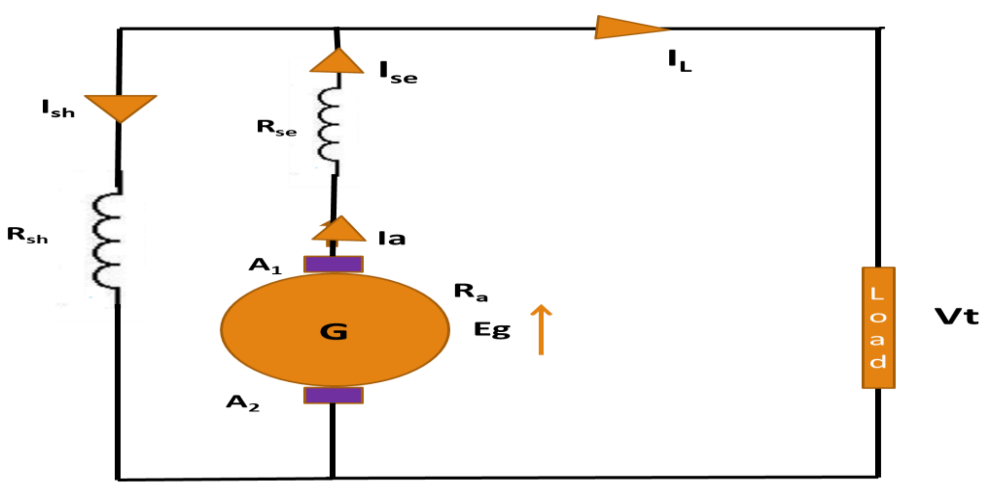
From the fig

Armature current $I_a = I_{se} + I_{sh}$ and $I_{se} = I_L$

Induced EMF $E_g = V_t + I_aR_a + I_{se}R_{se} + V_{brush}$

Terminal voltage $V_t = E_g - IaRa - I_{se} R_{se} - V_{brush}$

Long Shunt Compound generator: The shunt field winding is connected in parallel with the series combination of armature and series field winding.



From the fig

Armature current $I_a = I_L + I_{sh}$, $I_a = I_{se}$ and $I_{sh} = \frac{Vt}{Rsh}$

Induced EMF $E_g = V_t + I_aR_a + I_{se}R_{se} + V_{brush}$

$E_g = V_t + Ia(Ra + R_{se}) + V_{brush}$ $[I_a = I_{se}]$

Terminal voltage $V_t = E_g - Ia(Ra + R_{se}) - V_{brush}$

Problems:

1. **The emf generated in the armature of a shunt generator is 625 volts, delivering its full load current of 400 A to the external circuit. The field current is 6 amps and the armature resistance is 0.06Ω. What is the terminal voltage?**

Solution:

Given: Eg=625 V, I_L= 400 A, I_{sh} = 6A, and $R_a = 0.06\Omega$ Vt=?

Wkt $\mathbf{I_a = I_L + I_{sh}}$ = 400+6 = **406 A**

Terminal Voltage $\mathbf{V_t = E_g - I_aR_a}$ (neglecting brush voltage drop)

= 625 - (406X0.06)

V_t = 600.64V

2. **A 30 kW, 300V, DC shunt generator has armature and field resistances of 0.05 ohm and 100ohm respectively. Calculate the total power developed by armature when delivers full output power.**

Solution: P_L = 30kW, V_t = 300 V, $R_a = 0.05\Omega$, $R_{sh} = 100\Omega$ P_a= ?

Wkt the power developed in the armature $\mathbf{P_a = E_g \times I_a}$

Therefore P_L= **Vt x I_L**

$$I_L = \frac{PL}{Vt} = \frac{30\times10^3}{300} = \mathbf{100\ A}\ , \quad I_{sh} = \frac{Vt}{Rsh} = \frac{300}{100} = \mathbf{3\ A}$$

$\mathbf{I_a = I_L + I_{sh}}$ = 100 + 3 = **103A**

$E_g = V_t + I_a R_a$ = 300 + 103 X 0.05 = 305.15V

Power developed by armature = E_gI_a = 305.15 X 103 = 31.4304kW

3. A d.c. series generator has armature resistance of 0.5 Ω and a series field resistance of 0.03 Ω. It drives a load of 50 A. If it has 6 turns/coil and total 540 coils on the armature and is driven at 1500r.p.m., calculate the terminal voltage at the load. Assume 4 poles, lap type winding, flux per pole as 2mWb and total brush drop as 2 V.

Solution:

$R_a = 0.5\Omega$, $R_{se} = 0.03\Omega$, $I_L = 50A$ Total coils are 540 with 6 turns/coil.

i.e. Total turns = 540X6 = 3240

Total Conductors Z = 2 X Turns

$$Z = 2 \times 3240 = \mathbf{6480}$$

$N = 1500$ rpm **V_t= ?**

For P=4 lap type, A = P =4 and $\emptyset = 2$ m Wb = 2×10^{-3} Wb

$$E_g = \frac{\emptyset PNZ}{60A} = \frac{2X10^{-3} \times 4 \, X \, 1500 \times 6480}{60X \, 4} = \mathbf{324 \, V}$$

Wkt terminal voltage **$V_t = E_g - Ia(Ra + R_{se}) - V_{brush}$**

Where $I_a = I_L = 50A$

$$V_t = 324 - 50(0.5+0.03) - 2$$

$$\mathbf{V_t = 295.5V}$$

DC Motors

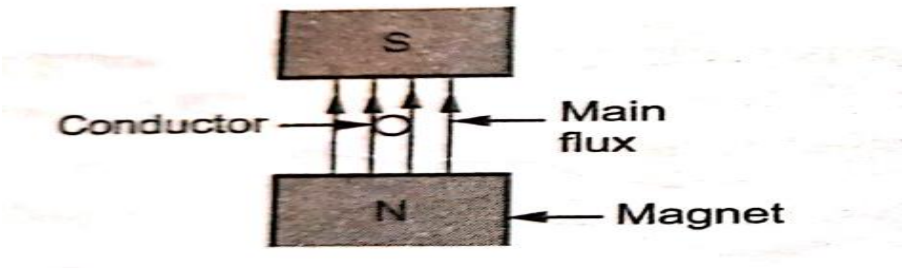
Operation of a DC motor:

- When a DC machine is loaded as a motor, the armature conductors carry current. These conductors lie in the magnetic field of the air gap. Thus, each conductor experiences a force. The conductors lie near the surface of the rotor at a common radius from its centre. Hence, a torque is produced around the circumference of the rotor, and the rotor starts rotating.

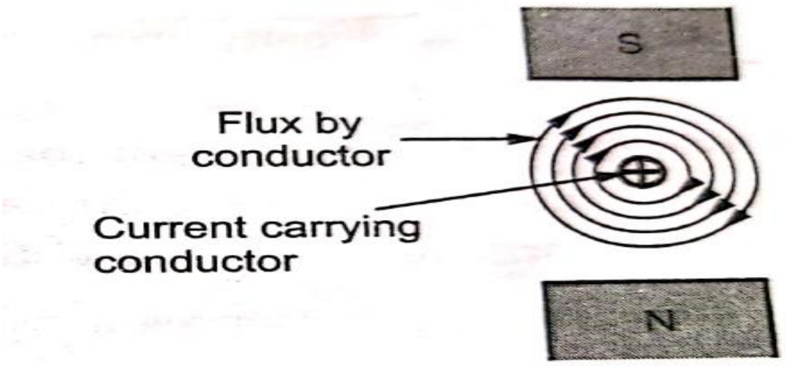
Working Principle of a DC motor

The principle of operation of the D C motor is" when a current carrying conductor is placed in a magnetic field, it experiences a mechanical force".

Consider a single conductor place in a magnetic field as shown in the fig and the main flux produced by the poles.



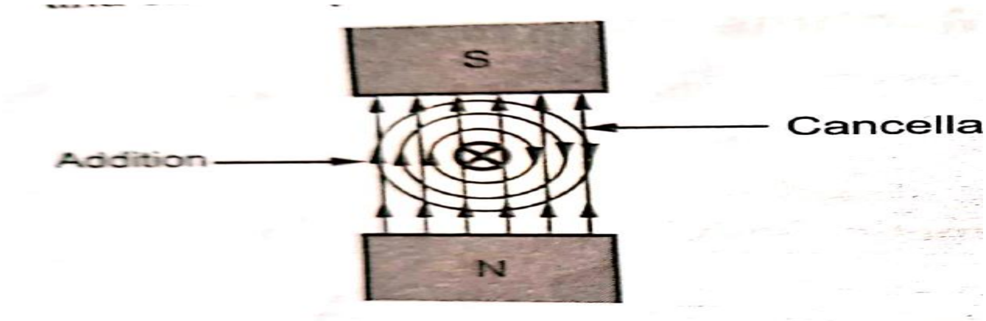
(a) Conductor in a magnetic field



When conductor excited by a separate supply it carries a current in a particular direction. Consider the conductor carries the current away from an observer as shown in the fig.

Any current carrying conductor produces its own magnetic field around it hence, this conductor also produces its own flux around it. The direction of this flux can be determined by right hand thumb rule. It is observed that the direction of flux is in clockwise direction.

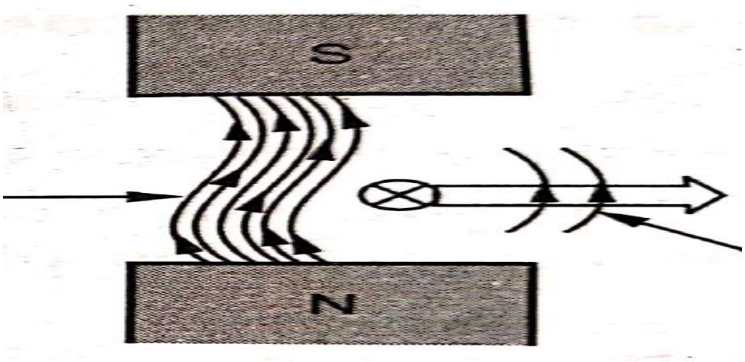
Now there are two fluxes present,
1. The flux produced by the poles called main flux.
2. The flux produced by the current carrying conductor.
These are shown in the fig



From this, it is clear that on one side (left side) of the conductor, both fluxes are in the same direction, there is gathering of the flux lines as two fluxes help each other.

As against this, on the right of the conductor, the two fluxes are in the opposite direction and hence try to cancel each other. Due to this, density of the flux line in this area gets weakened.

So on the left, there exists high flux density area while on the right of the conductor there exists low flux density as shown in the fig.



This flux distribution around the conductor acts like a stretched rubber band under tension. This exerts a mechanical force on the conductor which acts from high flux density area towards low flux density area, i.e. From left to right for the case considered as shown in the fig,

Due to this, overall armature experience a twisting force called torque and armature of the motor starts rotating.
The magnitude of the force experienced by the conductor in a motor is given by,

F=B l I Newtons(N)

B=Flux density due to the flux produced by the field winding.

l=Active length of the conductor.

I=Magnitude of the current passing through the conductor.

The direction of such force i.e. the direction of rotation of motor can be determined by **Fleming's right hand rule**.

Back Emf and its Significance:

- When the Armature of D C motor starts rotating and armature conductor cuts the magnetic flux, hence an EMF is induced in the Conductor called **Back EMF**.
- The induced emf acts in opposite direction to the applied voltage 'V'(Lenz's law) , hence it is called as back EMF. It is given by

$$E_b = \frac{\emptyset PNZ}{60\,A}$$

The Voltage equation of DC motor is **V = Eb + IaRa**

Therefore armature current $$I_a = \frac{V - E_b}{R_a}$$

Significance:

- The basic Principle of the Back EMF is that $E_b \propto N$
- When the load suddenly put on the motor, motor tries to slow down. So speed of the motor reduces due to which the back EMF decreases. So the net Voltage $(V - E_b)$ increases and motor draws **more armature current**.
- When the load on the motor decreases, the speed of the motor increases due to which the back EMF increases. So the net Voltage $(V - E_b)$ decreases and motor draws **less armature current**
- Therefore due to the presence of back emf. The d.c. motor acts as a self-regulating machine. It regulates the flow of armature current i.e., it automatically changes the armature current to meet the load requirement

Voltage equation of a Dc motor:

$$V = E_b + I_a R_a \quad \ldots\ldots..(1)$$

Multiplying the equation (1) by I_a we get

$$VI_a = E_b I_a + I_a^2 R_a \quad \ldots\ldots..(2)$$

Where,

VI_a is the electrical power input to the armature.

$I^2_a R_a$ is the copper loss in the armature.

$E_b I_a$ is the Mechanical power developed by the armature

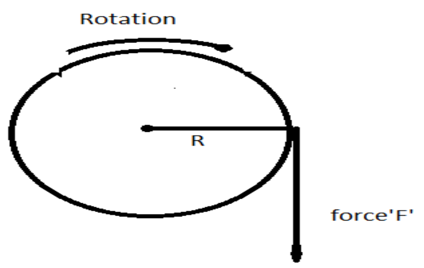
The mechanical power developed by the armature is Pm.

$$P_m = E_b I_a$$

Torque equation of a DC Motor:

The turning and twisting force about an axis is called **torque.**

Consider a wheel of radius 'R' meters acted upon the circumferential force 'F' netwons as shown in fig



The wheel is rotating with speed of 'N' rpm then its angular speed is,

$$\omega = \frac{2\pi N}{60} \text{ rad/sec} \qquad \text{--------1}$$

so work done in one revolution is

W= force X distance travelled in one revolution = **F X 2πR joules**

$$\textbf{Power} \quad P = \frac{workdone}{time\ for\ 1\ revolution} = \frac{F * 2\pi R}{\frac{60}{N}} = F*R*\frac{2\pi N}{60}$$

$$P = T * \omega$$

Where T =Torque in Nm and ω= angular speed in rad/sec

Let 'T_a'is torque developed in the armature of the motor. It is also called as **armature torque**.

The gross mechanical power developed in the armature is '$E_b I_a$'

Power in armature = armature torque *ω

$$E_b I_a = Ta * \frac{2\pi N}{60}$$

But, $$E_b = \frac{\emptyset PNZ}{60\ A}$$

Therefore $$\frac{\emptyset PNZ}{60\ A} * I_a = T_a * \frac{2\pi N}{60}$$

So, the torque equation is given as

$$T_a = \frac{1}{2\pi} X \frac{\varphi Ia PZ}{A}$$

Types of DC Motors:

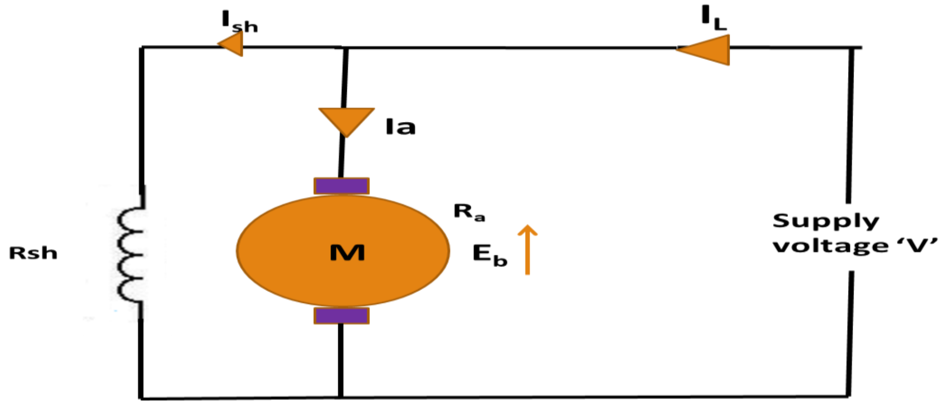
Motors are classified into 3 types: a) DC Shunt motor.

b) DC Series motor.

c) DC Compound motor.

a) DC Shunt motor:

- In shunt motor the field winding is connected in parallel with armature.
- The current through the shunt field winding is not the same as the armature current.



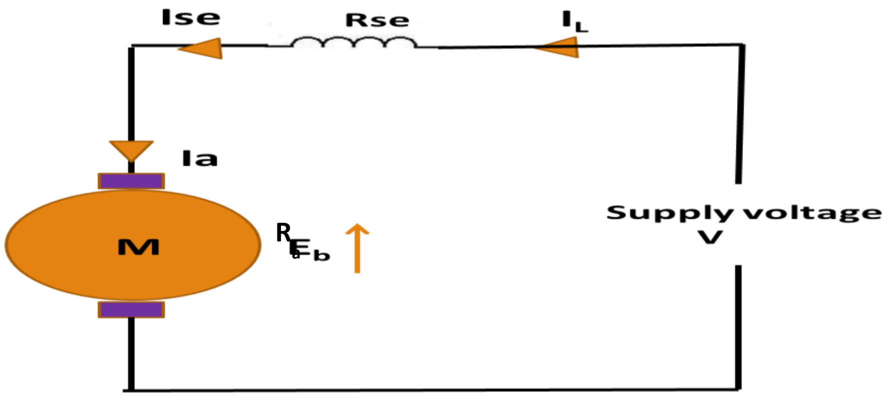
In above circuit

$$I_L = I_a + I_{sh} \quad \text{and} \quad I_{sh} = \frac{V}{Rsh}$$

$$\mathbf{V = E_b + I_aR_a + V_{brush}}$$

b) DC Series motor:

- In series wound motor the field winding is connected in series with the armature.
- Therefore, series field winding carries the armature current.



In above circuit

$$\mathbf{I_L = I_a = I_{se}}$$

$$\mathbf{V = E_b + I_aR_a + IseR_{se} + V_{brush}}$$

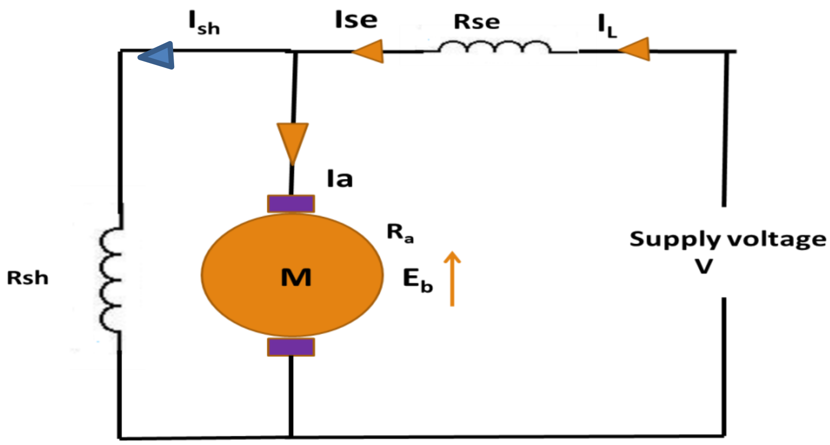
$$\mathbf{V = E_b + Ia(Ra + R_{se}) + V_{brush}}$$

c) DC Compound motor:

- Compound wound motor has two field windings; one connected in parallel with the armature and the other in series with it.
- There are two types of compound motor connections :

1) Short-shunt connection Compound Motor

When the shunt field winding is connected in parallel with the armature winding it is called short-shunt connection.



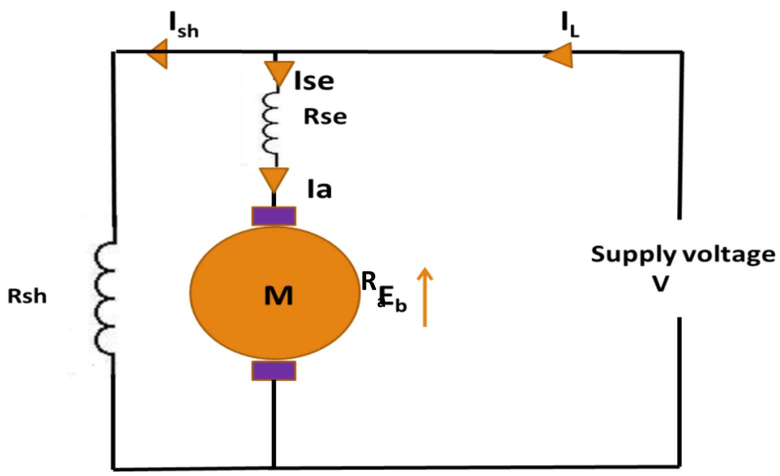
In above circuit

$$I_L = I_{se} = I_a + I_{sh}$$

$$V = E_b + I_a R_a + I_{se} R_{se} + V_{brush}$$

Long shunt connection Compound Motor

When the shunt winding is so connected that in parallel with the series combination of armature and series field it is called long-shunt connection



In above circuit

$I_L = I_{se} + I_{sh}$ and $I_{se} = I_a$

$$V = E_b + I_aR_a + IseR_{se} + V_{brush}$$

$$V = E_b + Ia(Ra + R_{se}) + V_{brush}$$

List of the formulas

1) $E_b = \frac{\emptyset PNZ}{60\ A}$

2) Armature torque $T_a = \frac{1}{2\pi} X \frac{\varphi IaPZ}{A}$

Shaft torque $T_{sh} = P / \omega$

3) The mechanical power developed by the armature is $\mathbf{P_m = E_bI_a}$

4) DC Shunt motor

$I_L = I_a + I_{sh}$ and $I_{sh} = \frac{V}{Rsh}$

$$V = E_b + I_aR_a + V_{brush}$$

5) **DC series Motor** $\mathbf{I_L = I_a = I_{se}}$

$$V = E_b + Ia(Ra + R_{se}) + V_{brush}$$

PROBLEMS

1. **A 4 pole DC motor takes 50A armature current. The armature has lap connected winding with 480 conductors. The flux per pole is 20mwb. Calculate the gross torque developed in the armature.**

Solution: Given P=4 lap onnected A = P = 4

I_a = 40A Z = 480 $\phi = 20 \times 10^{-3}$ wb T_a =?

Wkt Armature torque $T_a = \frac{1}{2\pi} X \frac{\varphi IaPZ}{A}$

$$T_a = \frac{1}{2\pi} X \frac{20 \times 10^{-3} \times 40 \times 4 \times 480}{4}$$

$$T_a = 76.39 \text{ N-m}$$

2. **A 200V, 4 pole, lap wound, d.c shunt motor has 800 conductors on its armature. The resistance of armature winding is 0.5 ohm & that of shunt field winding is 200 ohm. The motor takes a current of 21A, the flux/pole is 30mWb. Find the speed & gross torque developed in the motor**

Solution: Given V = 200 V P=4 lap connected A = P = 4

DC shunt motor Z = 800 $R_a = 0.5\Omega$ and $R_{sh} = 200\Omega$

$I_L = 21A$ $\phi = 30 \times 10^{-3}$ wb

N =? T_a =?

Wkt $E_b = \frac{\emptyset PNZ}{60\ A}$ $N = \frac{E_b x 60\ x A}{\emptyset PZ}$

For DC Shunt motor

$V = E_b + I_aR_a$ I,e $E_b = V - I_aR_a$

$I_L = I_a + I_{sh}$

$$I_{sh} = \frac{V}{Rsh} = \frac{200}{200} = 1\ A$$

$I_a = I_L - I_{sh} = 21 - 1 = 20A$

Therefore $E_b = V - I_aR_a = 200 - 20x0.5$

$E_b = 190V$

$$N = \frac{E_b x 60\ x A}{\emptyset PZ} = \frac{190 x 60\ x 4}{30 \times 10^{-3} x 4 x 800}$$

N= 475 rpm

Armature torque $T_a = \frac{1}{2\pi} X \frac{\varphi Ia PZ}{A}$

$$T_a = \frac{1}{2\pi} X \frac{30 \times 10^{-3} x\ 20 x\ 4\ x\ 800}{4}$$

T_a = 76.38 N-m

3. **A 4 pole, 220V , lap connected ,DC shunt motor has 36 slots, each slot has 16 conductors. It draws a current of 40A from the supply. The field and armature resistances are 110Ω and 0.1Ω respectively. The motor develop an output power of 6KW .the flux per pole is 40mwb calculate**
   - **i) The speed**
   - **ii) Torque developed in the armature**
   - **iii) Shaft torque**

Solution: Given V = 220 V P=4 lap connected A = P = 4

DC shunt motor Z = 36 x16 =576

I_L = 40A R_a= 0.1Ω and R_{sh}= 110Ω

P = $6\ x10^3$ W ϕ = 40 x 10^{-3} wb

N =? T_a =? T_{sh} =?

Wkt $E_b = \frac{\emptyset PNZ}{60\ A}$ $N = \frac{E_b x60\ xA}{\emptyset PZ}$

For DC Shunt motor

$V = E_b + I_aR_a$ I,e $E_b = V - I_aR_a$

$I_L = I_a + I_{sh}$

$I_{sh} = \frac{V}{Rsh} = \frac{220}{110} = 2A$

$I_a = I_L - I_{sh}$ = 40 -2 =38A

Therefore $E_b = V - I_aR_a$ = 220 – 38x0.1

E_b =216.2 V

$N = \frac{E_b x60\ xA}{\emptyset PZ} = \frac{216.2x60\ x4}{40 \times 10^{-3} x4x576}$

N= 563.02 rpm

Armature torque $T_a = \frac{1}{2\pi} X \frac{\varphi Ia PZ}{A}$

$$T_a = \frac{1}{2\pi} X \frac{40 \times 10^{-3} \times 38 \times 4 \times 576}{4}$$

$$T_a = 139.207 \text{ N-m}$$

Shaft torque $T_{sh} = \frac{P}{\omega}$

$$= \frac{6 \times 10^3}{2\pi N/60}$$

$$= \frac{6 \times 10^3 \times 60}{2\pi \times 563.02} = 101.73 \text{ N-m}$$

4. **220 V series motor is taking a current of 40A, resistance of armature 0.5 ohms, resistance of series field is 0.25 ohms. Calculate**
   i) **Back Emf**
   ii) **Power wasted in armature, and power wasted in series field.**

Solution: Given $V = 220$

DC Series motor $I_L = 40A$ $R_a = 0.5\Omega$ **and** $R_{se} = 0.25\Omega$

$E_b = ?$ $P_a = ?$ $P_{se} = ?$

Wkt DC series Motor $I_L = I_a = I_{se} = 40A$

$$V = E_b + Ia(Ra + R_{se})$$

Therefore

$$E_b = V - Ia(Ra + R_{se})$$

$$E_b = 220 - 40(0.5 + 0.25)$$

$$E_b = 190 \text{ V}$$

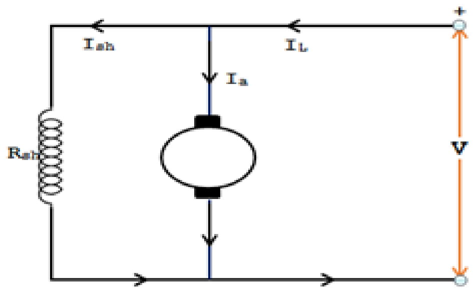
Power wasted in armature $P_a = I_a^2 R_a = 40^2 \text{ X } 0.5 = 800 \text{ W}$

Power wasted in series field $P_{se} = I_{se}^2 R_{se} = 40^2 \text{ X } 0.25 = 400 \text{ W}$

Characteristics of DC Motors:

The three important characteristic curves are

1. Torque V_s Armature current characteristic (T_a/I_a)
2. Speed V_s Armature current characteristic (N/I_a)
3. Speed V_s Torque characteristic (N/T_a)
DC Shunt Motor Characteristics:



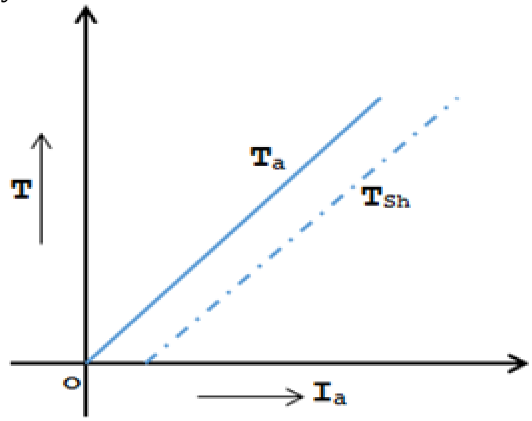
In DC shunt motor the field winding is connected in parallel with the source voltage, so the field current **Ish** and the **flux** are constant in a shunt motor .

Torque V_s Armature current characteristic (T_a/I_a):

We know that in a DC Motor $T_a \alpha \Phi I_a$ by torque equation

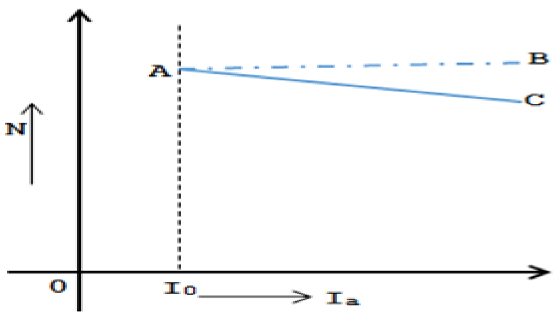
The flux Φ is constant in shunt motor, therefore $T_a \alpha I_a$

The torque increases linearly with the armature current

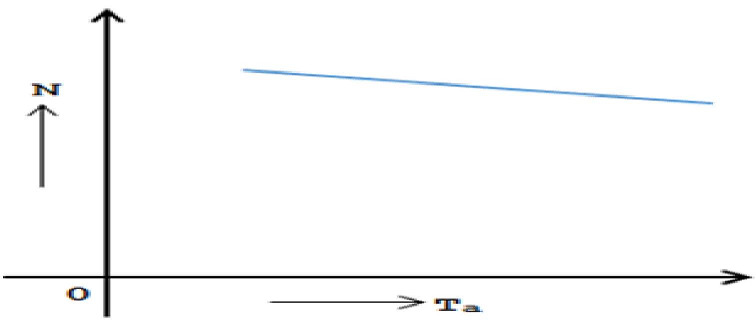


Speed V_s Armature current characteristic (N/I_a):

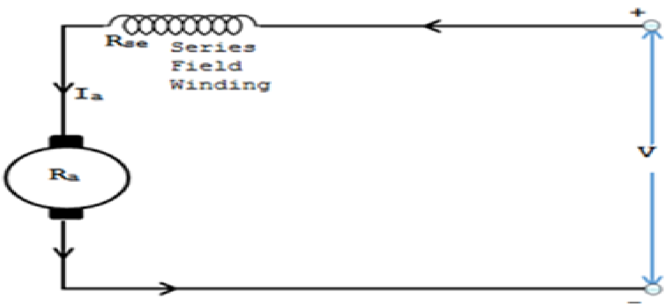
- $N \alpha E_b$ **and Eb=V-IaRa**. As the flux is constant.
- When load increases , the armature current increases hence the drop I_aR_a increases therefore **V-IaRa** decreases hence speed decreases.



Speed V_s Torque characteristic (N/T_a): The speed reduces when the load torque increases.



DC Series Motor:

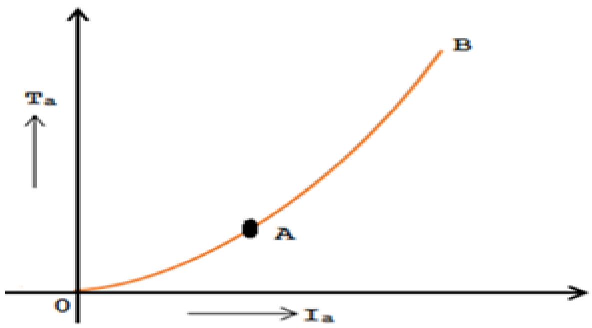


In DC series motor the field winding is connected in series with the source voltage, so the field current **Ise** and the **flux** are not constant.

Torque V_s Armature current characteristic (T_a/I_a):

We know that $Ta \propto \Phi Ia$

$$\mathbf{Ta \propto I^2 a}$$



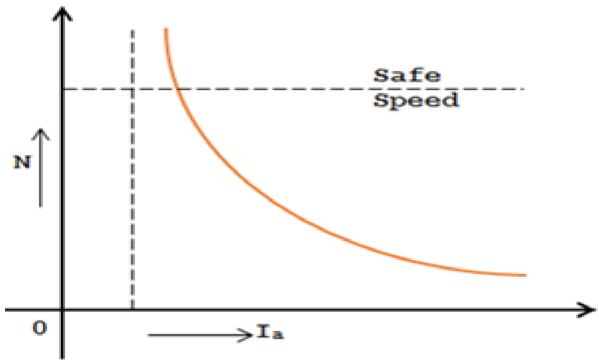
- The armature torque vs. armature current curve up to magnetic saturation is a parabola, which is shown in the characteristic curve OA.
- On the other hand once the magnetic saturation is reached, the Ta is directly proportional to the Ia.
- As a result the armature torque vs. armature current magnetic saturation characteristic is a straight line, which is shown in the curve AB.

Speed V_s Armature current characteristic (N/I_a):

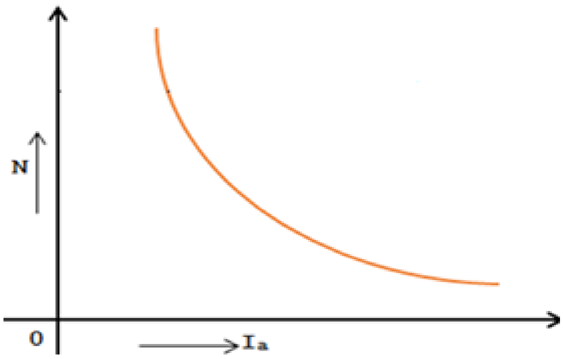
In Series Motor Speed -- $N \alpha (E_b/\Phi)$

$$N \alpha \ 1/ I_a \phi$$

$$N \alpha \ 1/ I_a^2$$



Speed V_s Torque characteristic (N/T_a): The speed reduces when the load torque increases.



Necessity of a Starter:

The starter is not required to start a DC. Motor but it enables us to start the motor in desired, safe way.

At the starting instant the speed of the motor is zero, (N = 0) and back emf E_b =0

The voltage equation of a de motor is, **$V = E_b + I_a R_a$**

At start $V = I_a R_a$ Therefore $I_a = V / R_a$

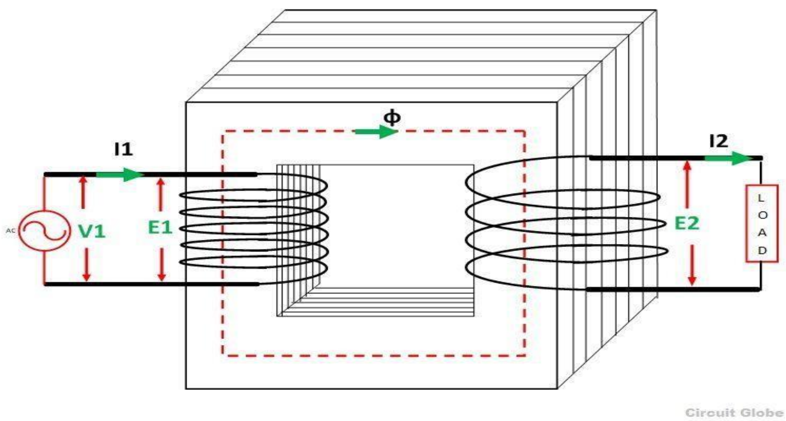
So at start, motor is showing a tendency to draw an armature current which may affect the performance of the motor and may burn out the winding

Module 3 (b): Single Phase Transformers

Introduction

- Transformer is a static device which transfer an electric power from one electrical circuit to another electrical circuit, with or without change of the voltage and without change of the frequency.

WORKING PRINCIPLE: - A transformer works on the principle of mutual induction between two magnetically coupled coils.



Let N1-----> be the number of turns in coil 1 and
N_2 ------> be the number of turns in coil 2

When the supply Voltage 'V_1' is applied to the coil 1 the current 'I_1' starts flowing in the winding, which sets an alternating flux 'φ'. Hence an emf 'E_1' induced in the coil 1 due the Electromagnetic Induction .

I,e $E_1 = -N_1 \frac{d\emptyset}{dt}$ (self Induced Emf)

The part of flux 'φ' links the coil 2,which induces an Emf 'E_2' in coil 2 due to Mutual Induction. Hence current 'I_2' starts flowing coil 2.

i.e $E_2 = -N_2 \frac{d\emptyset}{dt}$ (mutually Induced Emf) .

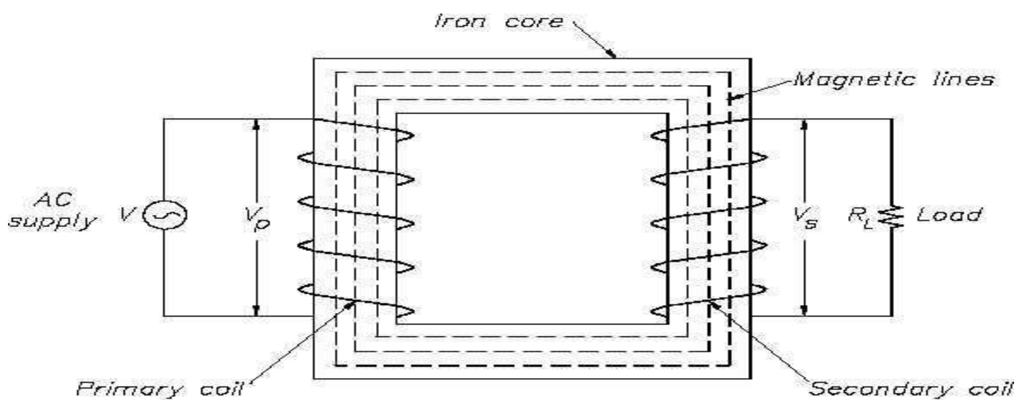
Therefore we will get output voltage 'V_2' across the coil 2.

CONSTRUCTION:

There are two basic parts of a transformer:

1) Magnetic core 2) winding

- The core of the transformer is either rectangular or square in size.
- The core is divided into i) Yoke ii)Limb
- Core is the Rectangular in shape which is made of thin sheets of silicon steel, which are laminated in order to reduce eddy current losses.
- The laminated sheets are overlapped so that to avoid air gap and they stamped together to form a core.
- The steel laminations are insulated from each other by using insulations like varnish
- The core provides low reluctance path for the flux provided by the winding
- The vertical portion on which the winding is wound is called Limb.
- The top and bottom horizontal portion is called Yoke.
- The core forms the magnetic circuit
- There are 2 windings i) Primary winding ii) Secondary winding which formsthe Electric circuit, made up of conducting material like copper.
- The winding which is connected to the supply is called primary winding andhaving 'N_1' number of turns.
- The winding which is connected to a load is secondary winding and having'N_2' number of turns.

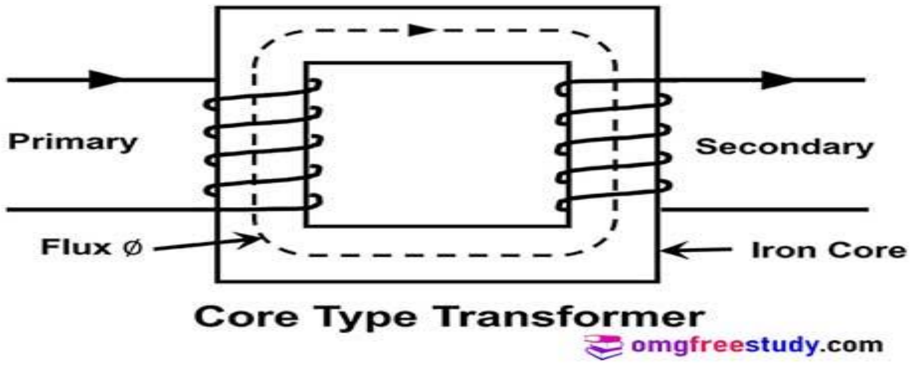


TYPES OF TRANSFORMER:

I) Based on Construction the transformer is divided into:
a) CORE TYPE b) SHELL TYPE.

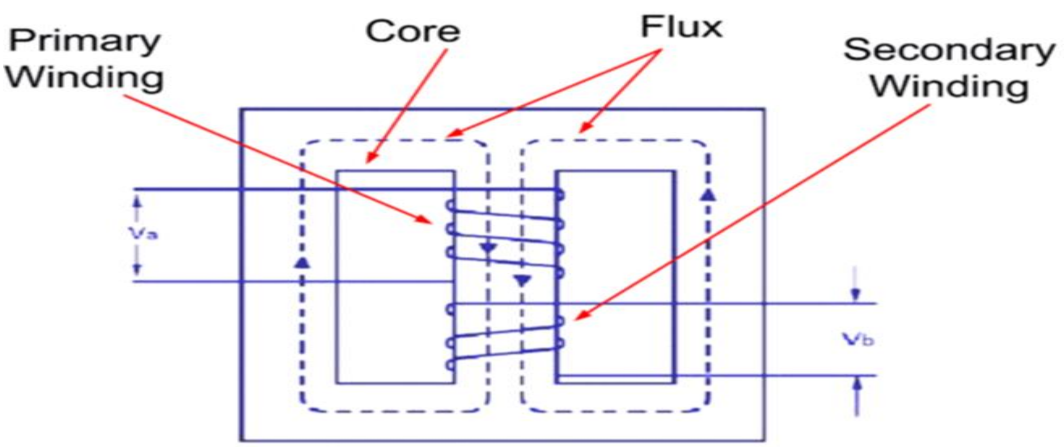
Core type transformer:

- It is rectangular in shape.
- It consists of 2 limbs on which the windings are wound.
- Since the windings are placed on the outer limbs of the core they can be easily removed for maintenance.
- The windings encircle the core.
- It has single magnetic circuit.
- It used for used for low voltage application.



Shell type transformer:

- It is rectangular in shape.
- It consists of 3 limbs and both the windings are wound on a central limb of the core.
- Since the windings are placed on the central limb of the core they cannot be easily removed for maintenance.
- The core encircles the winding.
- It has double magnetic circuit.
- It used for used for high voltage application



II) Based on of turns in primary and secondary winding the transformer is divided into:
a) STEP UP TRANSFORMER b) STEP DOWN TRANSFORMER c) ONE-ONE TRANSFORMER

Step up transformer

When N2 > N1 then (V2 > V1) the voltage is raised on the output side and is known as Step up transformer

Step down transformer

When N2 < N1 then (V2 < V1) the voltage level is lowered on the output side and is known as Step down transformer

(iii) 0ne-one transformer

When N2 = N1 then V2 =V1 the voltage is same on both side

Losses in a Transformer

The transformer has two types of losses

- Core or Iron losses [constant losses]
- Copper losses [cu losses]

Iron Losses (P_i)

Iron losses are caused by the alternating flux in the core of the transformer as this loss occurs in the core it is also known as Core loss. Iron loss is further divided into hysteresis and eddy current loss.
Once the core is manufactured the losses occurs in the core are constant. Hence the name constant losses

1. Hysteresis Loss

Due to the alternating flux setup in the core of the transformer, it undergoes a cycle of magnetization and demagnetization. Due to hysteresis effect there is a loss of energy in this process which is called hysteresis loss
The hysteresis loss can be minimized by using silicon steel material for the construction of core

2. Eddy Current Loss

The EMF induced in the winding sets up an eddy current in the core, the losses due to the eddy current is called eddy current losses.
The eddy current loss is minimized by making the core with thin laminations.

Copper losses (Pcu)

The copper losses are the power wasted in the form of I^2R loss due to the resistance of primary and secondary winding.
The copper loss depends on the magnitude of current flowing though the windings
When the load is connected across the transformer the current I_1 and I_2 starts flowing in the primary and secondary winding.
The losses can be minimized by designing the winding with low resistance conducting material
Thus total losses in the transformer = iron losses + copper losses
= Pi +Pcu

EMF equation of a single phase transformer

Let

N_1 - be the no. of turns of the primary winding

N_2 – be the no. of turns secondary winding

f – Frequency in Hz

Φ – flux in weber

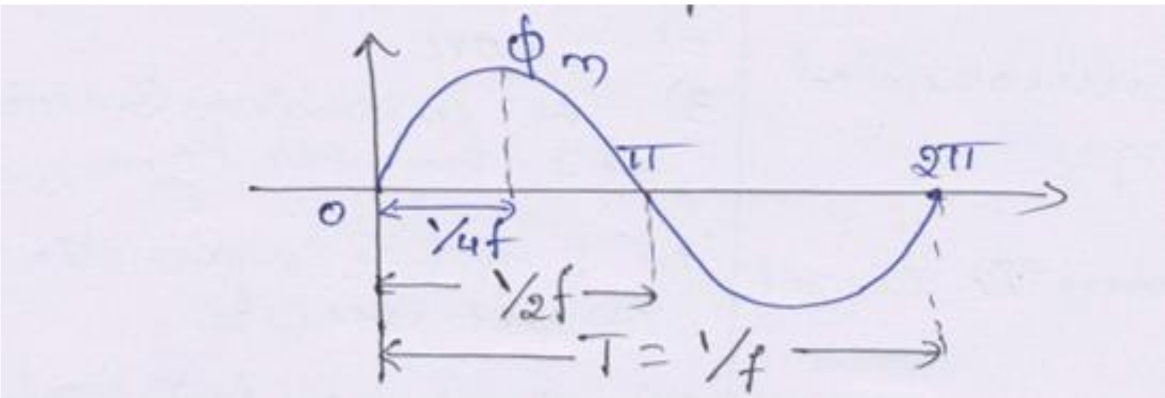
E_1- be the RMS value of emf induced in the primary

E_2 – be the RMS value of emf induced in the secondary

When the supply Voltage is applied to the primary winding which sets an alternating flux 'φ'. Hence an EMF 'E1' and 'E2' are induced in the primary and secondary winding respectively

$$e = \frac{d\emptyset}{dt} \quad \text{-------------- (i)}$$

Consider a one cycle of EMF



Let us consider 1/4th cycle of EMF

The change in flux in 1/4th cycle is

$d\phi = \Phi_m - 0 = \Phi_m$

The time taken to complete 1/4th of cycle is

dt = 1/4f

Substituting dϕ and dt in equation I we get

$$E_{avg} = \frac{d\phi}{dt} = \frac{\Phi m}{1/4f} = 4 f \Phi m$$

we know that $E_{rms} = 1.11 \times E_{avg}$

Therefore $E_{rms} = 1.11 \times 4 f \Phi m$

$E_{rms} = 4.44 f \Phi m$ **induced per turn**

If N_1 be the number of turns in primary then $E_1 = 4.44 f \Phi m N_1$ volts

If N_2 be the number of turns in secondary then $E_2 = 4.44 f \Phi m N_2$ volts

Transformer ratio

Voltage Ratio

W.k.t $E_1 = 4.44 f \varphi m N_1$

$E_2 = 4.44 f \varphi m N_2$

Taking a ratio $\frac{E_2}{E_1} = \frac{4.44 f \varphi m N_2}{4.44 f \varphi m N_1} = \frac{N_2}{N_1}$

The transformer rating is done in VA(volt ampere)

(power)VA rating of a transformer= $V_1I_1 = V_2I_2$

$$\frac{V_2}{V_1} = \frac{I_1}{I_2}$$

Therefore $\frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = K$(transformer ratio)

Efficiency

It is the ratio of the output power to the input power of a transformer

$$\eta = \frac{Power\ output}{Power\ Input}$$

Power input = Power output + losses

= Power output + P_{cu} + P_i

= $V_2I_2 \cos\varphi + P_{cu} + P_i$ wkt $P_{cu} = I^2R_2$

$$therfore\ \ \eta = \frac{V_2I_2\ \ \cos\emptyset}{V_2I_2\ \ \cos\emptyset + I_2^2\,R_2 + Pi} \times 100$$

Condition for Maximum efficiency

The efficiency of a transformer is given by

$$\eta = \frac{Power\ output}{Power\ Input}$$

Power input = Power output + losses

= Power output + P_{cu} + P_i

= $V_2I_2 \cos\varphi + P_{cu} + P_i$ wkt $P_{cu} = I^2R$

$$therfore\ \ \eta = \frac{V_2I_2\ \ \cos\emptyset}{V_2I_2\ \ \cos\emptyset + I_2^2\,R_2 + Pi} \times 100$$

Diff w.r.t I_2 and equating to zero

$$\frac{d\eta}{dI_2} = \frac{[V_2I_2\ \ \cos\emptyset + I_2^2\,R_2 + Pi]\,[V_2 \cos\emptyset] - V_2I_2\ \ \cos\emptyset\,[V_2 \cos\emptyset + 2I_2^2\,R_2]}{[V_2I_2\ \ \cos\emptyset + I_2^2\,R_2 + Pi]^2} = 0$$

$$[V_2I_2\ \ \cos\emptyset + I_2^2\,R_2 + Pi]\,\cancel{[V_2 \cos\emptyset]} = \cancel{V_2 \cos\emptyset}[I2[V_2 \cos\emptyset + 2I_2^2\,R_2]$$

$$[V_2I_2\ \ \cos\emptyset + I_2^2\,R_2 + Pi] = [V_2I_2 \cos\emptyset + 2I_2^2\,R_2]$$

$$\cancel{V_2I_2 \cos\emptyset} + I_2^2\,R_2 + Pi - \cancel{V_2I_2 \cos\emptyset} - 2I_2^2\,R_2 = 0$$

$$P_i - I_2^2\,R_2 = 0$$

Therefore $\mathbf{P_i = I_2^2\,R_2}$

$$\boxed{\mathbf{Pi = Pcu}}$$

List of formulas for problems

- $E1 = 4.44 f øm N1$
- $E2 = 4.44 f øm N2$
- Flux density Bm= φm/a wb/m^2
- Transformer ratio $k = \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2}$

  Power
- $Primary(VA\ rating) = V1I1 = E1I1$
- $Secondary\ (VA\ rating) = V2I2 = E2I2$
- I1= $\frac{VA\ rating}{V1}$ and I2= $\frac{VA\ rating}{V2}$
- **Efficiency** $$\%\,\eta = \frac{(VA\ rating)\ \cos\emptyset}{(VA\ rating)\ \cos\emptyset + P_i + I^2 R} \times 100$$

If n is fraction of load [n=1 for full load and n=0.5 for 50% (half)load etc **]**

$$\%\,\boldsymbol{\eta} = \frac{\boldsymbol{n}\ (\boldsymbol{VA\ rating})\ \boldsymbol{\cos\emptyset}}{\boldsymbol{n}(\boldsymbol{VA\ rating})\ \boldsymbol{\cos\emptyset} + \boldsymbol{P_i} + \boldsymbol{n^2 I^2} \mathbf{R}} \times \mathbf{100}$$

At maximum efficiency $P_i = P_{cu} = I^2 R$

Problems on EMF Equation & turns ratio

1) A single phase, 20KVA transformer has 1000 primary turns & 2500 secondary turns. The net cross sectional area of the core is 100 cm^2. When the primary winding is connected to 550V, 50HZ supply. Calculate i) the maximum value of the flux density in the core ii) The voltage induced in the secondary winding iii) the Primary & Secondary full currents.

Sol:- Given:- VA rating = 20KVA

$N_1 = 1000$ turns, $N_2 = 2500$ turns

$a = 100 cm^2 = 100 \times 10^{-4} m^2$

$E_1 = 550V$. $f = 50HZ$

i) $B_m = ?$ w.k.t $B_m = \phi_m / a$

w.k.t $E_1 = 4.44 f \phi_m N_1$

$550 = 4.44 \times 50 \times \phi_m \times 1000$

$$\phi_m = \frac{550}{4.44 \times 50 \times 1000} = 2.47 \times 10^{-3} wb$$

$$B_m = \frac{\phi_m}{a} = \frac{2.47 \times 10^{-3}}{100 \times 10^{-4}} = 0.247$$

$B_m = 0.247 \ wb/m^2$

2) $E_2 = ?$

w.k.t Voltage ratio is

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} \qquad \frac{E_2}{550} = \frac{2500}{1000}$$

$$E_2 = \frac{2500}{1000} \times 550 = 1375 \text{ Volts}$$

$$\boxed{E_2 = 1375 \text{ volts}}$$

3) $I_1 = ?$ $I_2 = ?$

w.k.t VA rating $= E_1 I_1$

$$20 \times 10^3 = 550 \times I_1$$

$$I_1 = \frac{20 \times 10^3}{550} = 36.36 \text{ A}$$

$$\boxed{I_1 = 36.36 \text{ A}}$$

VA rating $= E_2 I_2$

$$20 \times 10^3 = 1375 \times I_2$$

$$I_2 = \frac{20 \times 10^3}{1375} = 14.54 \text{ A}$$

$$\boxed{I_2 = 14.54 \text{ A}}$$

2) Find the numbers of turns on the primary & secondary side of a 440/230 V, 50 Hz single phase transformer. if the net area of cross section of the core is 30 cm² & flux density is 1 wb/m²

Sol:- Given:-

$$E_1 = 440V = V_1$$

$$E_2 = 230V = V_2$$

$$f = 50\,Hz, \quad a = 30\,cm^2 = 30\times10^{-4}\,m^2$$

$$B_m = 1\,wb/m^2$$

$$N_1 = ? \qquad N_2 = ?$$

w.k.t

$$E_1 = 4.44 f \phi_m N_1$$

$$\phi_m = B_m \cdot a = 1\times30\times10^{-4} = \underline{30\times10^{-4}\,wb}$$

$$\therefore \quad E_1 = 4.44 f \phi_m N_1$$

$$440 = 4.44\times50\times30\times10^{-4}\times N_1$$

$$N_1 = \frac{440}{4.44\times50\times30\times10^{-4}} = 660.6 = \underline{661}\text{ turns}$$

$$\boxed{N_1 = 661\text{ turns}}$$

w.k.t

$$E_2 = 4.44 f \phi_m N_2$$

$$230 = 4.44\times50\times30\times10^{-4}\times N_2$$

$$N_2 = \frac{230}{4.44 \times 50 \times 30 \times 10^{-4}} = 345.3$$

$$\boxed{N_2 = 345.34 \text{ turns}} = \underline{346 \text{ turns}}$$

3. A single phase transformer has 400 Primary & 1000 secondary turns. The net cross sectional area of core is $60 cm^2$. The Primary winding is connected to 500V, 50HZ. Find
i) peak value of core flux density
ii) EMf induced in the Secondary winding

Sol:- Given:- $N_1 = 400$ turns
$N_2 = 1000$ turns

$a = 60 cm^2 = \underline{60 \times 10^{-4} m^2}$

$E_1 = V_1 = 500V$ $f = 50V$

i) $B_m = ?$ w.k.t $B_m = \phi_m / a$

$E_1 = 4.44 f \phi_m N_1$

$500 = 4.44 \times 50 \times \phi_m \times 400$

$\phi_m = 500 / 4.44 \times 50 \times 400$

$\underline{\phi_m = 5.63 \times 10^{-3} \text{ wb}}$

$$B_m = \frac{\phi_m}{a} = \frac{5.63 \times 10^{-3}}{60 \times 10^{-4}}$$

$$\boxed{B_m = 0.938 \text{ wb}/m^2}$$

ii) $E_2 = ?$

w.k.t

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

$$\frac{E_2}{500} = \frac{1000}{400}$$

$$E_2 = \frac{1000}{400} \times 500$$

$$\boxed{E_2 = 1250V}$$

Problems on Efficiency (η).

4) Find the efficiency of 150KVA, single phase transformer at i) full load UPF
ii) 50% of load at 0.8 Pf
If the copper loss at full load is 1600 watts & Iron loss is 1400 watts

Sol:- Given:- VA rating = 150 KVA = 150×10^3 VA
$P_i = 1400$ watts $P_{cu} = 1600$ watts
$\cos\phi = UPF = 1$

i) η at full load = ?

** $n = 1$ for full load

w.k.t

$$\% \eta_{F.L} = \frac{(\text{VA rating}) \cos\phi}{(\text{VA rating}) \cos\phi + P_i + P_{cu}} \times 100$$

$$\%\eta_{F.L} = \frac{150\times10^{3}\times1}{150\times10^{3}\times1+1400+1600}\times100$$

$$\boxed{\%\eta_{F.L} = 98.03}$$

ii) $\%\eta_{H.L}$ at 50% of load = ? $\cos\phi = 0.8$

$n = 0.5$

$$\%\eta_{50\%} = \frac{n(\text{VA rating})\cos\phi}{n(\text{VA rating})\cos\phi + P_i + P_{cu}}\times100$$

$$= \frac{0.5\times150\times10^{3}\times0.8}{0.5\times150\times10^{3}\times0.8+1400+1600}\times100$$

$$\boxed{\%\eta_{50\%} = 95.23}$$

5) A 10KVA, 1ϕ transformer has a primary winding of 300turns & secondary winding of 750 turns, cross sectional area of 64 cm^2. If the primary voltage is 440 V at 50HZ, Find the maximum flux density in the core, Emf induced in secondary. At 0.8 lag P/f Calculate the efficiency of transformer if full load copper loss is 400W & iron loss is 200W.

Sol:- Given:- VA rating = 10 KVA = 10×10^{3} VA

N_1 = 300 turns

N_2 = 750 turns

$a = 64 cm^2 = 64\times10^{-4} m^2$

$E_1 = V_1 = 440V$, $f = 50 HZ$

$\cos\phi = 0.8$ P_{cu} = 400 watt, P_i = 200 watts

i) $B_m = ?$ when $B_m = \phi_m / a$.

w.k.t

$E_1 = 4.44 f \phi_m N_1$

$440 = 4.44\times50\times\phi_m\times300$

$$\phi_m = \frac{440}{4.44\times50\times300} = 6.6\times10^{-3} wb$$

$$B_m = \frac{\phi_m}{a} = \frac{6.6\times10^{-3}}{64\times10^{-4}} = 1.03\times10^{-8} wb/m^2$$

$$\boxed{B_m = 1.03\times10^{-8} wb/m^2}$$

ii) $E_2 = ?$

w.k.t $\frac{E_2}{E_1} = \frac{N_2}{N_1}$ $E_2 = \frac{750}{300}\times440$

$$\boxed{E_2 = 1100 \text{ volts}}$$

iii) $\%\eta = ?$ $\cos\phi = 0.8$ P_i = 200 watts

P_{cu} = 400 watts

$$\%\eta = \frac{(VA\ rating)\cos\phi}{(VA\ rating)\cos\phi + P_i + P_{cu}} \times 100$$

$$= \frac{10\times10^3 \times 0.8}{10\times10^3\times0.8 + 200 + 400} \times 100$$

$$\boxed{\%\eta = 93.02\%}$$

6) ** The maximum efficiency at full load & UPF of a single phase, 25KVA, 500/1000 V, 50HZ transformer is 98%. Determine the efficiency at i) 75% load 0.9 pf, ii) 50% load 0.8 pf iii) 25% load 0.6 pf.

Sol:- Given:- VA rating = 25 KVA = 25×10^3

$E_1 = 500V$, $E_2 = 1000V$, $f = 50HZ$

$\%\eta_{max} = 98\% = 0.98$ $\cos\phi$ = UPF = 1

w.k.t at full load

$$\eta_{max} = \frac{(VA\ rating)\times\cos\phi}{(VA\ rating)\cos\phi + P_i + P_{cu}}$$

At max eff $P_i = P_{cu}$

$$\therefore\ 0.98 = \frac{25\times10^3\times1}{25\times10^3\times1 + P_i + P_i}$$

$$0.98 = \frac{25\times10^3}{25\times10^3 + 2P_i}$$

$$0.98\left[25\times10^3 + 2P_i\right] = 25\times10^3$$

$$2P_i = 510.2$$

$$\boxed{P_i = 510.2 = P_{cu}}$$

i) $\eta_{75\%} = ?$ at $\cos\phi = 0.9$

$n = 0.75$

$$\eta_{75\%} = \frac{n(\text{VA rating})\times\cos\phi}{n(\text{VA rating}) + P_i + n^2 P_{cu}}$$

$$\eta_{75\%} = \frac{0.75\times250\times10^3\times0.9}{0.75\times250\times10^3\times0.9 + 510.2 + (0.75)^2\times510.2}$$

$$\boxed{\eta_{75\%} = 97.69\%}$$

ii) $\eta_{50\%} = ?$ at $\cos\phi = 0.8 \rightarrow n = 0.5$

$$\eta_{50\%} = \frac{0.5\times25\times10^3\times0.8}{0.5\times25\times10^3\times0.8 + 510.2 + (0.5)^2\times510.2}$$

$$\boxed{\eta_{25\%} = 96.98\%}$$

iii) $\eta_{25\%} = ?$ → $\cos\phi = 0.6$ → $n = 0.25$

$$\eta_{25\%} = \frac{0.25 \times 25 \times 10^3 \times 0.6}{0.25 \times 25 \times 10^3 \times 0.6 + 510.2 + (0.25)^2 \times 510.2} \times 100$$

$$\boxed{\eta_{25\%} = 95.9\%}$$