MODULE - 4

ENGINEERING ECONOMY

INTRODUCTION TO BASIC ECONOMICS

The word Economics has been derived from two Greek words, namely. Oikus and Nemein. Oikus means "household" and Nemein ineans "management".

Economics is the science that deals with the production and consumption of goods and services and the distribution and rendering of these for human welfare. The following life the economic goals.

- A high level of employment
- Price stability
- Efficiency
- An equitable distribution of income
- Growth

Management of money is critical to the success of any company. Therefore, a knowledge of the role money plays in the day to day operations of a construction company is of the utmost importance to a construction manager. Issues such as the borrowing of money, expenditure of money for expansion of company operations. Construction firms acquire funds, as most individuals do, by borrowing money from banks and similar lending resources. It is typical for instance, to borrow money to finance operations in the field pending receipt of progress payments from the client.

Interest and its application over time impact the cost of any transaction involving the lending, borrowing, and investment of money. In fact, the cost of money and_ the time during which money is tied up in any business decision process are crucial financial management factors.

CONCEPT OF ENGINEERING ECONOMICS

Science is a field of study where the basic principles of different physical systems are formulated and tested. Engineering is the application of science. It establishes varied application systems based on different scientific principles. Price has a major role in deciding the demand and supply of a product. Hence, from the organization's point of view, efficient and effective functioning of the organization would certainly help it to provide goods/services at a lower cost which in turn will enable it to fix a lower price for its goods or services

BASIC ECONOMIC PROBLEMS

1. Allocation of resources:

The wants or requirements of an individual, business firm or the government far exceed the resources available to satisfy these needs. So with scarce resources, a choice has to be made regarding which all wants should be satisfied. This allocation of resources has different objectives in different countries.

2. By what methods are goods produced:

Goods are produced, in various ways. For example, agricultural goods can be produced by using more of labour and less of capital or more capital and less of labor. When the former is used, it is known as labour intensive method while the latter is known as capital intensive method.

3. Distribution of goods

After the goods are produced, the question arises as to who is going to receive these and in what proportion.

4. Utilization of resources:

The next question is arc the resources of the country, such as Men, material and money, fully utilized or are they underutilized. This leads us to the question of employment of resources. When resources remain unuti]ized, goods and services do not get produced the cost of which is very huge for the society to hear. The resources should not only be put to use, but also efficient utilization of these is necessary.

5. Consumption:

Consumption means the use of resources or wealth in order to satisfy one's needs. It also means destruction of resources by human beings.

6. Production:

Consumption is only possible when there is production. As consumption is destruction of utilities similarly production is creation of utilities to satisfy human wants. Production is possible by the combination of four factors: land, labour, capital, and organization.

7. Exchange and Distribution:

Distribution means distribution of national income among various factors of production. The land owners get rent, labourers get wages, suppliers of money get rate of interest and organizers get profit.

8. Money and Banking:

Money is generally accepted by all as a means of payment, medium of exchange, standard of deferred payment, and store of all values.

9. Modern economy is based on money.

Economic activities are measured in terms of money. Bank is the financial institution that accepts money in the form of deposits and lends it for productive purposes.

10. International Trade:

When nations trade among themselves, it is denoted as international trade. No country can produce all the commodities it requires. So in order to satisfy various needs a large number of items are depending on international trade.

Definition of Engineering Economics

- Engineering economics deals with the methods that enable one to make economic decisions towards evaluation of design and engineering alternatives. It helps in examining the relevancy of a project, estimating its value and justifying it from the engineering viewpoint.
- Engineering economics provides methods that enable one to take economic decisions towards minimizing costs and/or maximizing benefits to business organizations.

Engineers use the knowledge of engineering economy in performing analysis, synthesizing and drawing conclusions as they work on projects of all sizes. The success of engineering and business projects is normally measured in terms of financial efficiency. A project will be able to achieve maximum financial efficiency when it is properly planned and operated with respect to its technical, social, and financial requirements. Since engineers understand the technical requirements of a project, they can combine the technical details of the project and the knowledge of engineering economy to perform economy study to arrive at a sound managerial decision.

PROBLEM SOLVING AND DECISION MAKING

An engineering economist draws upon the accumulated knowledge of engineering and economics to fashion and employ tools to identify a preferred course of action. There is still considerable debate about their theoretical bases and how they should be used. There are many aspects to consider and many ways to consider them.

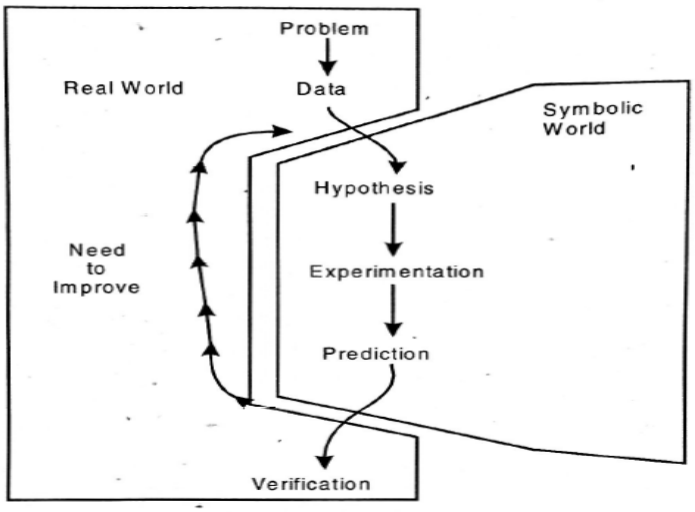

Fig. 4.1. Problem solving process

The fundamental approach to economic problem solving is to elaborate on the time honoured scientific method.

The method. is anchored in two worlds: the real, everyday working world and the abstract, scientifically oriented world as shown in Fig. 4.1.

Problems in engineering and managerial economy originate in the real world of economic planning, management, and control. The problem is confined and clarified by data from the real world. This information is combined with scientific principles supplied by the analyst to formulate a hypothesis is symbolic terms. By manipulating and experimenting with the abstractions of the real world, the analyst can simulate multiple configurations of reality that otherwise would be too costly on too inconvenient to investigate.

From this activity a prediction usually emerges. The predicted behaviour is converted back to reality for testing in the form of hardware designs, or commands. If it is valid, the problem is solved. If not, the cycle is repeated with the added information that the previous approach was unsuccessful.

PRINCIPLES OF ENGINEERING ECONOMY

The following are seven principles of Engineering Economics.

Principle 1 : Develop the alternatives

The choice (decision) is among the alternatives. The alternatives are to be identified and then defined for subsequent analysis. A decision situation involves making a choice among two or more alternatives. Developing and defining the alternatives for direct evaluation is important because of the resulting impact on the quality of the decision.

Principle 2: Focus on the differences

Only the difference in expected future outcomes among the alternatives is relevant to their comparison and should be considered when making the decision. If all prospective outcomes of the feasible alternatives were exactly the same, then there would be no basis or need for comparison. We would be indifferent to the alternatives and make decision on the basis of random selection.

Principle 3: Use a consistent viewpoint

The prospective outcomes of the alternatives, economic and other, should be consistently developed from a defined viewpoint (perspective). It is important that the viewpoint for a particular decision be first defined and then used consistently in the description, analysis and comparison of the alternative.

Principle 4: Use a common unit of measure

Using a common unit of measurement to enumerate as many of the prospective outcomes as possible will make easier the analysis and comparison of alternatives.

Principle 5: Consider all relevant criteria

Selection of a preferred alternative (decision-making requires the use of a criterion or several criteria). The decision process should consider both the outcomes enumerated in the monetary unit and those expressed in some other unit of measurement made explicit in a descriptive manner.

Principle 6: Make uncertainty explicit

Uncertainty is inherent in projecting for estimating the future outcomes of the alternative recognized m their analysis and comparison.

Principal 7: Revisit your decision

Improved decision-making results from an adoptive process. To the extent practicable, the projected outcomes of the selected alternative should he subsequently compared with the actual results achieved.

- Problem recognition, definition and evaluation
- Development of feasible alternatives
- Development of cash flow for each alternative
- Selection of criteria
- Analysis and comparison of the altemativ.es
- Selection of the preferred alternative
- Performance monitoring and post-evaluation results

MICROECONOMICS AND MACROECONOMICS

The subject matter of Economics has been divided into two parts Microeconomics and Macroeconomics.

Microeconomics deals with the analysis of individual units and small groups of individual units such as individual income, price and demand for a product, supply of a goods, etc. Two conditions of the mixed economy that are most important for microeconomics, including efficiency, and equity, that are generally desired by society and pursued by governments throi1gh economic policies

Macroeconomics is the study of economics system as a whole. It deals with the study of aggregates covering the entire economy such as national income, national product, general price level, employment, aggregate demand, aggregate supply, and so on.

Macroeconomics is focused on the movement and trends in the economy as a whole, while in microeconomics the focus is placed on factors that affect the decisions made by firms and individuals. The factors that are studied by macro and micro will often influence each other, such as the current level of unemployment in the economy as a whole will affect the supply of workers.

DIFFERENCE BETWEEN MICROECONOMICS AND MACROECONOMICS

S.No.	Basis of	Microeconomics	Macroeconomics
1.	Definition	Microeconomics if the study of particular firm, particular household, individual price, wage, income, industry and particular commodity.	Macroeconomics is concerns with such variables as the aggregate volume of the output of an economy, with the extent to which its resources are employed, with the size of national income and with the general price level.
2.	Objectives	The objective of microeconomics is optimum allocations of resources.	The objective of macroeconomics is full employment and evelopment of economic resources.
3.	Demand depends	Consumer's expectations and the price of the particular product.	Household's expectations and the price of the all products.
4.	Supply depends	Expectations of profits by firms and the price of the good or services	Producer's expectations and total productions costs
5.	Nature of activity	Microeconomics is based on disaggregation.	Microeconomics is based on aggregation
6.	Assumptions	Microeconomics is assumed that there is full employment.	Macroeconomics is assumed that the allocation of resources is constant.
7.	Equilibrium	Equilibrium occurs when the quantity demanded equals the quantity supplied.	Equilibrium in an economy occurs when the aggregate demand equals the aggregate supply.
8.	Price	There is a price for each good or service that will clear the market.	There is a price level in an economy at which the aggregate demand will equal aggregate supply.

DEFINITIONS FOR ENGINEERING ECONOMIC TERMS

- **Present Worth**

Present worth, Present Value and Principal all represent the value of money at time zero, which is the beginning of the engineering economic analysis period under investigation. In formulas, the present sum of money may be labelled as PW, PV, P or P_0. All four of these symbols represent the same initial time frame, which is time zero.

- **Future Worth**

Future worth (FW), future value (F) or (F0) represent the future sum of money including principal plus interest. Future values occur at any point in time in the future and they are usually designated as the end of the engineering economic analysis period if they are the last activity to occur in the analysis period. The future worth of present values, and payments and disbursement streams, includes interest on the money interested or withdrawn from an account.

- **Annuities: Uniform Series**

Annuities represent a payment or disbursement stream deposited or withdrawn at equal set intervals such as daily, weekly, monthly, or yearly. As each annuity is deposited into an interest bearing account, it begins to draw interest at the end of each compounding period. The annuities deposited, plus any previous interest earned, are used when calculating the interest on the funds in the account at the end of each period.

Annuity(A) is characterized by

- Equal payment
- Equal periods between payments and
- The first payment occurring at the end of the first period.

A series of payments made at the beginning instead at the end of each period is referred to as annuity due. In this case, calculation will be slightly different from general annuity. It will differ in the following ways

- The series should be divided into two equal parts.
- The first payment should be treated separately.
- The remaining payments should follow the rule of general annuity calculation.

- **Salvage value**

The salvage value is what an asset is worth at the end: of its useful life. In engineering economic analysis, the salvage value is represented by a future value occurring at the end of the analysis period. It is not always possible to accurately determine what a future salvage value of an asset will be; therefore, for the purpose of an analysis, a reasonable salvage value is assumed and included in the calculations. Many times, salvage values for similar items from previous projects are incorporated into a new analysis.

- **Sunk Cost**

Sunk cost is a difficult concept to understand when performing engineering economic analysis. Sunk cost represents funds not recoverable because they have already been expended sometime in the past. This is known as the past cost of an equipment/asset.

- **Marginal Cost**

Marginal cost of a product is the cost of producing an additional unit of that product.

- **Marginal Revenue**

Marginal revenue of a product is the incremental revenue of selling an additional unit of that product.

- **Opportunity Cost**

In practice, if an alternative (A) is selected from a set of competing alternatives (A,B), then the corresponding investment in the selected alternative is not available for any other purpose. If the same money is invested in some other alternative (B), it may fetch some return. Since the money is invested in the selected alternative (A), one has to forego the return from the other alternative (B). The amount that is foregone by not investing in the other alternative (B) is known as the opportunity cost of the selected alternative (A). So the opportunity cost of an alternative is the return that will be foregone by not investing the same money in another alternative.

- **Capitalized Cost**

Capitalized cost is a term used in engineering economics and it refers to the present worth of a project with an infinite life. In other words, capitalized cost is a lump sum of money needed today (t = 0) to support an infinite life project simply on earned interest only. The concept of capitalized cost usually applies to public projects such as airports, bridges, dams, and long-term private projects such as hospitals and private airports. Since most present value interest factors are the same after 50 to 100 years depending on interest rates, the concept of perpetual annuity may be used to detem1ine the present worth of infinite life projects as capitalized cost.

Since it is difficult to calculate the capitalized cost of a project with a stream of infinite cash flows when they vary from year to year or they occur irregularly, it is necessary first to convert those cash flows to a uniform series or annuity and then use the perpetual annuity concept to calculate the capitalized cost of that project.

INTEREST FORMULAS

Interest rate can be classified into simple interest rate and compound interest rate. In simple interest, the interest is calculated, based on the initial deposit for every interest period. In this case, calculation of interest on interest is not applicable. In compound interest, the interest for the current period is computed based on the amount (principal plus interest up to the end of the previous period) at the beginning of the current period. The notations which are used in various interest formulae are as follows:

- P = principal amount
- n = No. of interest periods
- i = interest rate (It may be compounded monthly, quarterly, semiannually or annually)
- F = future amount at the end of year n
- A= equal amount deposited at the end of every interest period
- G = uniform amount which will be added/subtracted period after period to/ from the amount of deposit A 1 at the end of period 1

TIME VALUE OF MONEY

The time value of money is important when one is interested either in investing or borrowing the money. If a person invests his money today in bank savings, by next year he will definitely accumulate more money than his investment. This accumulation of money over a specified time period is called as time value of money. The time value of money is generally expressed by interest amount. The original investment or the borrowed amount (i.e. loan) is known as the principal. The amount of interest indicates the increase between principal amount invested or borrowed and the final amount received or owed.

In case of an investment made in the past, the total amount of interest accumulated till now is given by total amount to be received- original investment (i.e., principal amount). Similarly, in case of a loan taken in past, the total amount of interest is given by amount of interest = Present amount owed - original loan (i.e., principal amount)

In both the cases there is a net increase over the amount of money that was originally invested or borrowed.

When the interest amount is expressed as the percentage of the original amount per unit time, the resulting parameter is known as the rate of interest and is generally designated as 'i'.

The time period over which the interest rate is expressed is known as the interest period. The interest rate is generally expressed per unit year. However, in some cases the interest rate may also be expressed per unit month.

Example 4.1

A person deposited t 1,50,000 in a bank for one year and got t 1,60.000 at the end of one year. Find out the total amount of interest and the rate of interest per year on the deposited money.

Solution:

The total amount of interest gained over one year·=

1,60,000 - 1,50,000 = 10,000 The rate of interest 'i' per year is given by

$$i(\%) = \frac{₹\ 10{,}000}{₹\ 1{,}50{,}000} \times 100 = 6.67\%$$

4.14 SIMPLE INTEREST

The interest is said to simple, when the interest is charged only on the principal amount for the interest period. No interest is charged on the interest amount accrued during the preceding interest periods. In case of simple interest, the total amount of interest accumulated for a given interest period is simply a product of the principal amount, the rate of interest and the number of interest periods. It is given by the following expression.

$$I_T = \frac{P \times n \times i}{100}$$

Where

I_T = total amount of interest

P = Principal amount

n = number of interest periods

i = rate of interest (%)

Simple interest reflects the effect of time value of money only on the principal amount.

4.15 COMPOUND INTEREST

The interest is said to be compound, when the interest for any interest period is charged on principal amount plus the interest amount accrued in all the previous interest periods. Compound interest takes into account the effect of time value of money on both principal as well as on the accrued interest also.

$$I_T = \mathrm{P}\left(1 + \frac{i}{1000}\right)^n$$

The following example will explain the difference between the simple and the compound interest.

Example: 4.2

A person has taken a loan of amount of ₹ 1,00,000 from a bank for a period of 5 years. Estimate the amount of money, the person will repay to the bank at the end of 5 years for the following cases;

a) Considering simple interest rate of 8% per year

b) Considering compound interest rate of 8% per year.

Solution :

(a) Considering the simple interest @ 8% per year;

The interest for each year = 1,00,000 × 1 × 0.08 = ₹ 8000.

The interest for each is year is calculated only on the principal amount i.e., ₹ 1,00,000.

Thus the interest accumulated at the end of each year is constant i.e., ₹ 8000.

The year - by - year details about the interest accrued and amount owed at the end of each year are shown in Table 4.1.

Table 4.1 Payment using simple interest

End of year (EOY)	Amount of interest (₹)	Total amount owed (₹)
1	8000	1,08,000
2	8000	1,16,000
3	8000	1,24,000
4	8000	1,32,000
5	8000	**1,40,000**

(b) Considering the compound interest @ 8% per year;

The amount of interest and the total amount owed at the end of each year, considering compound interest are presented in Table 4.2.

$$\text{Interest amount} = P\left(1+\frac{i}{100}\right)^n$$

$$\text{End of first year} = 1{,}00{,}000\left(1+\frac{8}{100}\right)^1 = 1{,}08{,}000$$

$$\text{End of second year} = 1{,}00{,}000\left(1+\frac{8}{100}\right)^2 = 1{,}16{,}640$$

Table 4.2 Payment using simple interest

End of year (EOY)	Amount of interest (₹)	Total amount owed (₹)
1	8,000	1,08,000
2	8,640	1,16,640
3	9,331	1,25,971
4	10,078	1,36,049
5	10,883	1,46,932

From these calculations it is clear that, in case of compound interest the interest for each year is calculated on the principal amount plus the interest amount accumulated till that period.

4.16 CASH FLOW DIAGRAM

The graphical representation of the cash flows i.e. both cash outflows and cash inflows with respect to a time scale is generally referred as cash flow diagram. A typical cash flow diagram is shown in Fig. 4.2. The cash flows are generally indicated by vertical arrows on the time scale as shown in Fig. 4.2. The cash outflows (i.e., costs or expense) are generally represented by vertically downward arrows whereas the cash inflows (i.e. revenue or income) are represented by vertically upward arrows. In the cash flow diagram, number of interest periods is shown on the time scale. The interest period may be a quarter, a month or a year. Since the cash flows generally occur at different time intervals within an interest period, for ease of calculation, all the cash flows are assumed to occur at the end of an interest period. Thus in Fig. 4.2, the numbers on the time scale represent the end of year (EOY).

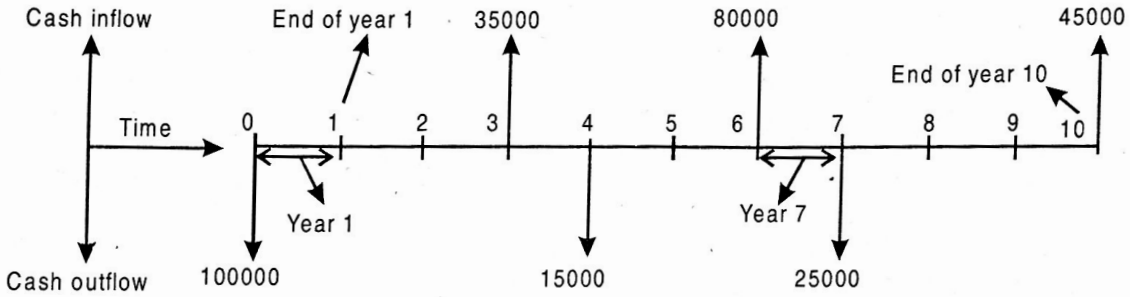

Fig. 4.2 : Cash flow diagram

In Fig. 4.2 the cash outflows are ₹ 100000, ₹ 15000 and ₹ 25000 occurring at end of year (EOY) '0' i.e. at the beginning, EOY '4' and EOY '7' respectively. Similarly the cash inflows ₹ 35000, ₹ 80000 and ₹ 45000 are occurring at EOY '3', EOY '6' and EOY '10' respectively.

4.17 SINGLE-PAYMENT

4.17.1 Single-payment compound amount

Here, the objective is to find the single future sum (F) of the initial payment (P) made at time 0 after n periods at an interest rate *i* compounded every period. The cash flow diagram of this situation is shown in Fig. 4.3.

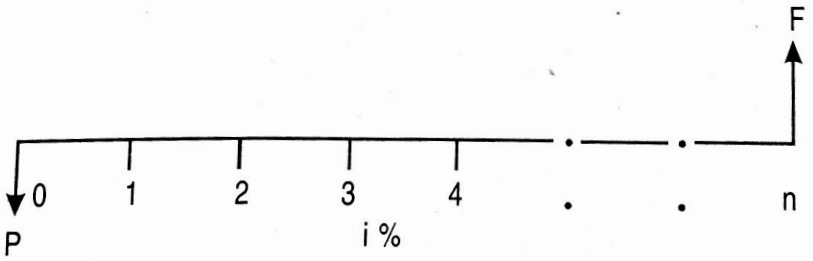

Fig. 4.3 : Cash flow diagram of single-payment compound amount

The formula to obtain the single-payment compound amount is

$$F = P(1 + i)^n = P(F/P, i, n)$$

where

$(F/P, i, n)$ is called as single-payment compound amount factor.

Example: 4.3

A person deposits a sum of ₹ 40,000 at the interest rate of 15% compounded annually for 10 years. Find the maturity value after 10 years.

Solution :

$P = ₹\ 40{,}000$

$i = 15\%$ compounded annually

$n = 10$ years

$F = P(1 + i)^n = P(F/P, i, n)$ $\quad | = (1 + i)^n$

$= 40{,}000\ (F/P, 15\%, 10)$ $\quad | = (1 + 0.15)^{10}$

$= 40{,}000 \times 4.04 = ₹\ 1{,}61{,}600$ $\quad | = 4.04$

The maturity value of ₹ 40,000 invested now at 15% compounded yearly is equal to ₹ 1,61,600 after 10 years.

4.17.2 Single-Payment Present Worth Amount

Here, the objective is to find the present worth amount (P) of a single future sum (F) which will be received after n periods at an interest rate of i compounded at the end of every interest period. The corresponding cash flow diagram is shown in Fig. 4.4.

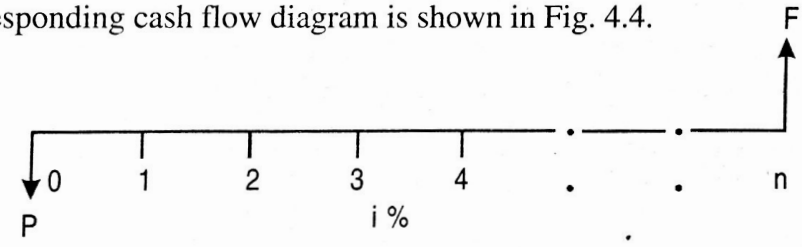

Fig. 4.4 : Cash flow diagram of single-payment present worth amount

The formula to obtain the present worth is

$$P = \frac{F}{(1+i)^n} = F(P/F, i, n)$$

where

$(P/F, i, n)$ is termed as single-payment persent worth factor.

Example: 4.4

A person wishes to have a future sum of ₹ 1,50,000 for his son's education after 10 years from now. What is the single-payment that he should deposit now so that he gets the desired amount after 10 years? The bank gives 13% interest rate compounded annually.

Solution :

F = ₹ 1,50,000

i = 13%, compounded annually

n = 10 years

$P = F/(1 + i)^n = F(P/F, i, n)$

= 1,50,000 (P/F, 13%, 10)

$$\frac{1}{(1+i)^n} = \frac{1}{(1+0.13)^{10}} = 0.295$$

= 1,50,000 × 0.295

= ₹ 44,250

The person has to invest ₹ 44,250 now so that he will get a sum of ₹ 1,50,000 after 10 years at 13% interest rate compounded annually.

4.18 EQUAL PAYMENT

4.18.1 Equal-Payment Series Compound Amount

In this type of investment mode, the objective is to find the future worth of n equal payments which are made at the end of every interest period till the end of the n^{th} interest period at an interest rate of i compounded at the end of each interest period. The corresponding cash flow diagram is shown in Fig. 4.5.

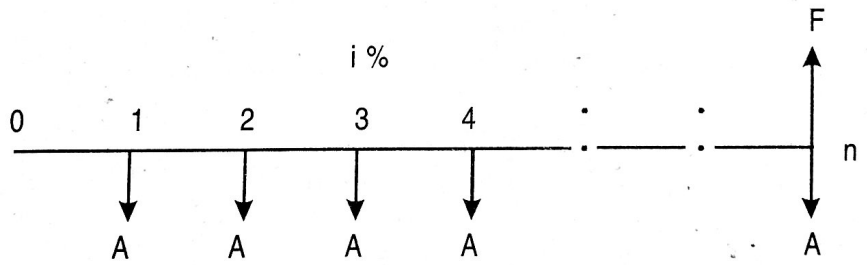

Fig. 4.5 : Cash flow diagram of equal-payment series compound amount

In Fig. 4.5,

A = equal amount deposited at the end of each interest period

n = No. of interest periods

i = rate of interest

F = single future amount

The formula to get F is

$$F = A\frac{(1+i)^n - 1}{i} = A(F/A, i, n)$$

where

$(F/A, i, n)$ is termed as *equal-payment series compound amount factor.*

Example 4.5

A person who is now 30 years old is planning for his retired life. He plans to invest an equal sum of ₹ 20,000 at the end of every year for the next 30 years starting from the end of the next year. The bank gives 15% interest rate, compounded annually. Find the maturity value of his account when he is 60 years old.

Solution :

$A = ₹\ 20{,}000$

$n = 30$ years

$i = 15\%$

The corresponding cash flow diagram is shown in Fig. 4.6.

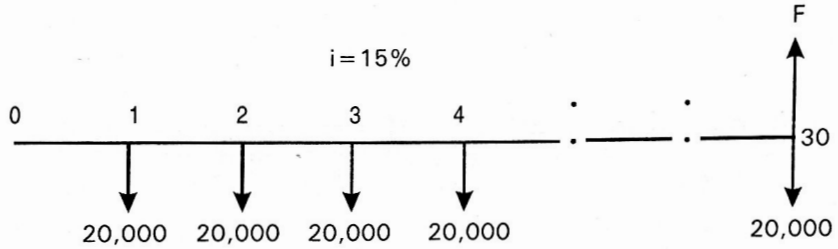

Fig. 4.6 : Cash flow diagram of equal - payment series compound amount

$$F = A\frac{(1+i)^n - 1}{i} = A(F/A, i, n)$$

$$\frac{(1+i)^n - 1}{i}$$

$$= 20{,}000\ (F/A,\ 15\%,\ 30)$$

$$= \frac{(1+0.15)^{30} - 1}{0.15}$$

$$= 20{,}000 \times 434.74$$

$$= ₹\ 86{,}94{,}800$$

The future sum of the annual equal payments after 30 years is equal to ₹ 86,94,800.

4.18.2 Equal-Payment Series Sinking Fund

In this type of investment mode, the objective is to find the equivalent amount (A) that should be deposited at the end of every interest period for n interest periods to realize a future sum (F) at the end of the nth interest period at an interest rate of i. The corresponding cash flow diagram is shown in Fig. 4.7.

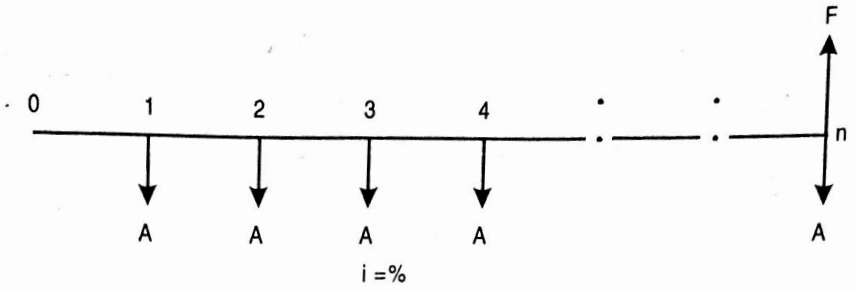

Fig. 4.7 : Cash flow diagram of equal-payment series compound amount

In Fig. 4.7,

A = equal amount to be deposited at the end of each interest period

n = No. of interest periods

i = rate of interest

F = single future amount at the end of the n^{th} period

The formula to get F is

$$A = F\frac{i}{(1+i)^n - 1} = F(A/F, i, n)$$

where

$(A/F, i, n)$ is called as *equal-payment series sinking fund factor.*

Example 4.6

A company has to replace a present facility after 10 years at an outlay of ₹ 5,00,000. It plans to deposit an equal amount at the end of every year for the next 10 years at an interest rate of 15% compounded annually. Find the equivalent amount that must be deposited at the end of every year for the next 10 years.

Solution :

F = ₹ 5,00,000

n = 10 years

i = 15%

The corresponding cash flow diagram is shown in Fig. 4.8.

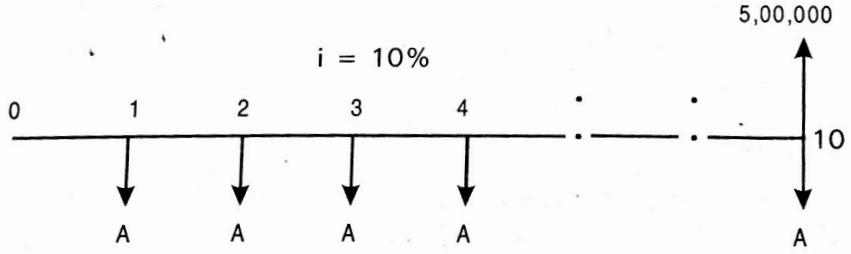

Fig. 4.8 : Cash flow diagram of equal - payment series sinking fund

$$A = F\frac{i}{(1+i)^n - 1} = F(A/F, i, n)$$

$$= 5{,}00{,}000\ (A/F, 15\%, 10)$$

$$= 5{,}00{,}000 \times 0.049$$

$$= ₹\ 24{,}500$$

$$\frac{i}{(1+i)^n - 1}$$

$$\frac{0.15}{(1+0.15)^{10} - 1} = 0.049$$

The annual equal amount which must be deposited for 10 years is ₹ 24,500.

4.18.3 Equal-Payment Series Present Worth Amount

The objective of this mode of investment is to find the present worth of an equal payment made at the end of every interest period for n interest periods at an interest rate of i compounded at the end of every interest period.

The corresponding cash flow diagram is shown in Fig. 4.9. Here,

P = present worth

A = annual equivalent payment

i = interest rate

n = No. of interest periods

The formula to compute P is

$$P = A\frac{(1+i)^n - 1}{i(1+i)^n} = A(P/A, i, n)$$

where

$(P/A, i, n)$ is called *equal - payment series present worth factor.*

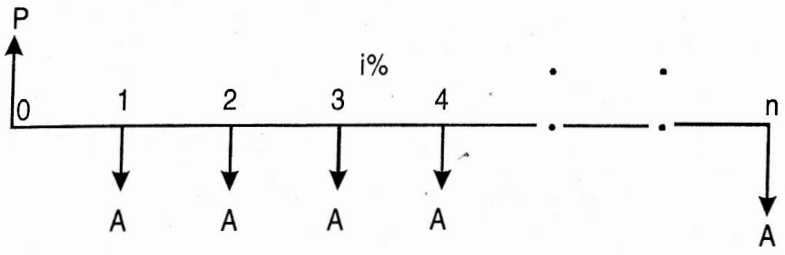

Fig. 4.9 : Cash flow diagram of single-payment series present worth amount

4.18.4 Equal-Payment Series Capital Recovery Amount

The objective of this mode of investment is to find the annual equivalent amount (A) which is to be recovered at the end of every interest period for n interest periods for a loan (P) which is sanctioned now at an interest rate of i compounded at the end of every interest period (Fig. 4.10).

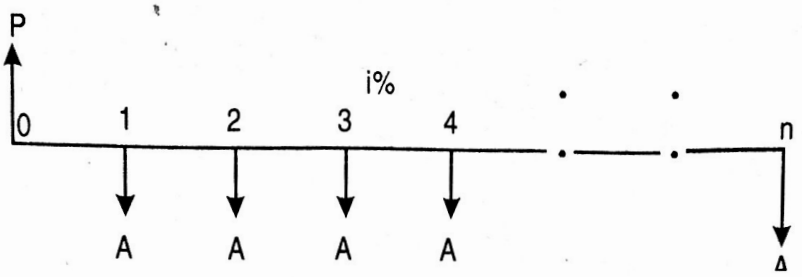

Fig. 4.10 : Cash flow diagram of equal-payment series present recovery amount

In Fig. 4.10,

P = present worth (loan amount)

A = annual equivalent payment (recovery amount)

i = interest rate

n = No. of interest periods

The formula to compute A is as follows:

$$A = P\frac{i(1+i)^n}{(1+i)^n - 1} = P(A/P,i,n)$$

where

$(A/P, i, n)$ is called *equal - payment series capital recovery factor.*

4.19 UNIFORM GRADIENT SERIES

4.19.1 Uniform Gradient Series Annual Equivalent Amount

The objective of this mode of investment is to find the annual equivalent amount of a series with an amount $A1$ at the end of the first year and with an equal increment (G) at the end of each of the following $n - 1$ years with an interest rate i compounded annually.

The corresponding cash flow diagram is shown in Fig. 4.11.

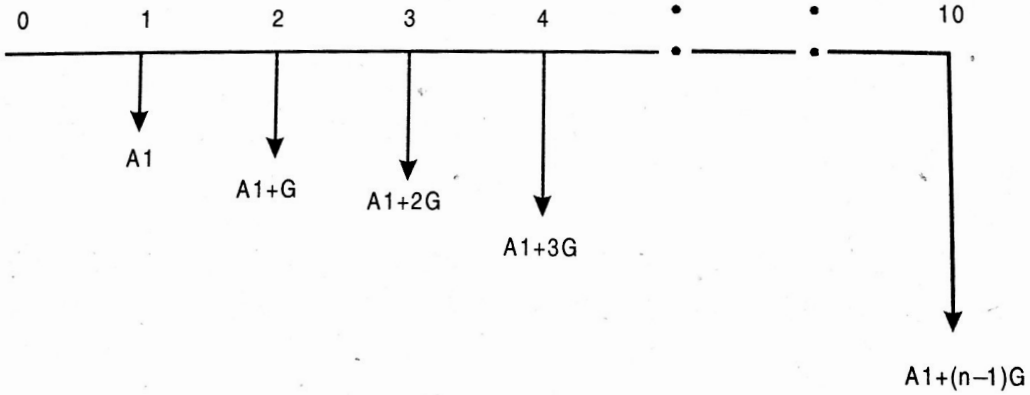

Fig. 4.11 : Cash flow diagram of equal-payment series sinking fund

The formula to compute A is

$$A = A1 + G\frac{(1+i)^n - in - 1}{i(1+i)^n - i}$$

$$= A1 + G(A/G, i, n)$$

where

$(A/G, i, n)$ is called *uniform gradient series factor.*

4.20 NOMINAL AND EFFECTIVE INTEREST RATES

The interest or discount rate in all of the engineering economic formulas is the discount rate per time period. Typically interest or discount rates are quoted on an annual basis. These rates are termed nominal interest or discount rates; nominal rates also specify the compounding that occurs during the year. A discount rate that is 12% compounded monthly is an example of a nominal interest rate. The actual interest or effective interest rate (on an annual basis) is greater than the nominal interest rate because of the compounding that occurs during the year, i.e., the nominal and effective rates are equal only when the compounding period is a year.

Nominal Interest Rate (NIR) is the periodic interest rate times the number of periods per year. NIR does not take the effect of the compounding during a year into account whereas. Effective Interest Rate (EIR) does.

Effective Interest Rate

Let i be the nominal interest rate compounded annually. But, in practice, the compounding may occur less than a year. For example, compounding may be monthly, quarterly, or semi-annually. Compounding monthly means that the interest is computed at the end of every month. There are 12 interest periods in a year if the interest is compounded monthly. Under such situations, the formula to compute the effective interest rate, which is compounded annually, is Effective interest rate,

$$R = \left(1 + \frac{i}{C}\right)^C - 1$$

where,

i = the nominal interest rate

C = the number of interest periods in a year.

Example 4.7

A person invests a sum of ₹ 10,000 in a bank at a nominal interest rate of 10% for 10 years. The compounding is quarterly. Find the maturity amount of the deposit after 10 years.

Solution :

P = ₹ 10,000

n = 10 years

$i = 10\%$ (Nominal interest rate)

No. of interest periods per year, $C = 4$

Effective interest rate, $R = (1 + i/C)^C - 1$

$= 0.1038$

$= (1 + 10\%/4)^4 - 1$

$= 10.38\%$, compounded annually.

$F = P(1 + R)^n = 10{,}000(1 + 0.1038)^{10}$

$= ₹\ 26{,}847.50$

4.21 DEFERRED ANNUITY

If the occurrence of annuity does not happen until some later date, the annuity is known as deferred annuity. In other words, when a uniform series begins at a time other than the end of period 1, it is called a shifted uniform series. Several methods can be used to determine the equivalent present worth P for the case of deferred annuity. For example, P of the uniform series shown in Fig. 4.12 can be determined by any one of the following methods.

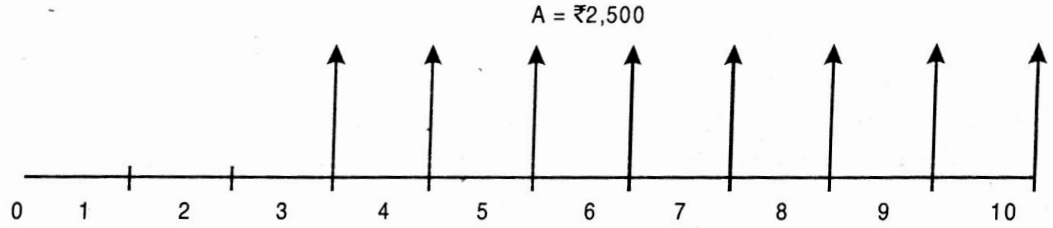

Fig. 4.12 : Cash flow diagram of shifted uniform series

In case of deferred annuity, the first payment does not begin until some date later than the end of the first period. Deferred annuity can be calculated by dividing the series into two equal parts.

- The first part is the number of payments paid, which follows general annuity calculation.
- The second part is number of periods.
- Find out the present worth of annuity, then discount this value through the pre-annuity period. Hence, we get the deferred annuity.

4.22 COMPARISON OF ALTERNATIVES

For most of the engineering projects, equipments etc., there are more than one feasible alternative. It is the duty of the project management team (comprising of engineers, designers, project managers etc.) of the client organization to select the best alternative that involves less cost and results more revenue. For this purpose, the economic comparison of the alternatives is made. The different cost elements and other parameters to be considered while making the economic comparison of the alternatives are initial cost, annual operating and maintenance cost, annual income or receipts, expected salvage value, income tax benefit and the useful life. When only one, among the feasible alternatives is selected, the alternatives are said to be mutually exclusive.

4.22.1 Comparison of alternatives by present worth method

In the present worth method for comparison of mutually exclusive alternatives, the future amounts i.e. expenditures and incomes occurring at future periods of time are converted into equivalent present worth values at a certain rate of interest per interest period and are added to present worth occurring at '0' time. The converted equivalent present worth values are always less than the respective future amounts since the rate of interest is normally greater than zero. The cash flow of the mutually exclusive alternatives may consist of future expenditures and incomes in different forms namely randomly placed single amounts, uniform amount series commencing from end of year 1, randomly placed uniform amount series i.e. commencing at time period other than end of year 1, positive and negative uniform gradient series starting either from end of year 1 or at different time periods and geometric gradient series etc. The different compound interest factors namely single payment present worth factor, uniform series present worth factor and present worth factors for arithmetic and geometric gradient series etc. will be used to convert the respective future amounts to the equivalent present worth values for different alternatives.

Example: 4.8

Two alternatives for purchase of transit mixer having same useful life proposed.

Alternative-1: Initial purchase cost = ₹6,00,000, Annual operating and maintenance cost = ₹40,000, Expected salvage value = ₹2,50,000, Useful life = 5 years.

Alternative-2: Initial purchase cost = ₹4,00,000, Annual operating and maintenance cost = ₹70,000, Expected salvage value = ₹1,40,000, Useful life = 5 years. Using present worth method, find out which alternative should be selected, if the rate of interest is 10% per year.

Solution:

Since both alternatives have the same life span i.e. 5 years, the present worth of the alternatives will be compared over a period of 5 years.

The cash flow diagram of Alternative-1 is shown in Fig. 4.13.

The cash outflows i.e. costs or expenditures are represented by vertically downward arrows whereas the cash inflows i.e. revenue or income are represented by vertically upward arrows.

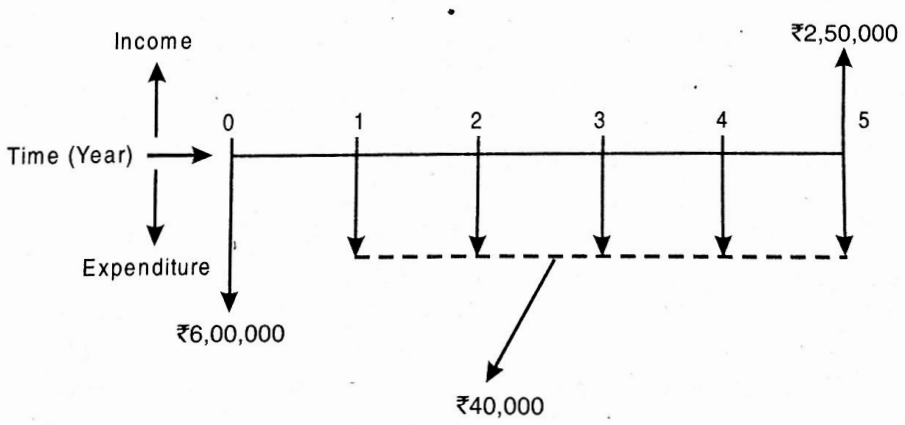

Fig. 4.13 : Cash flow diagram of Alternative-1

The equivalent present worth of Alternative-1 i.e. PW_1 is calculated as follows;

The initial cost, P = ₹6,00,000 (cash outflow),

Annual operating and maintenance cost, A = ₹40,000 (cash outflow),

Salvage value, F = ₹2,50,000 (cash inflow).

$PW_1 = -6,00,000 - 40,000(P/A, i, n) + 2,50,000(P/F, i, n)$

$PW_1 = -6,00,000 - 40,000(P/A, 10\%, 5) + 2,50,000(P/F, 10\%, 5)$

$$PW_1 = -6,00,000 - 40,000 \times \frac{(1+i)^n - 1}{i(1+i)^n} + 2,50,000 \times \frac{1}{(1+i)^n}$$

$$PW_1 = -6,00,000 - 40,000 \times \frac{(1+0.1)^5 - 1}{0.1(1+0.1)^5} + 2,50,000 \times \frac{1}{(1+0.1)^5}$$

$PW_1 = -6,00,000 - 40,000 \times 3.79 + 2,50,000 \times 0.62$

$\boldsymbol{PW_1 = -₹\ 5,96,600}$

The cash flow diagram of Alternative-2 is shown in Fig. 4.14.

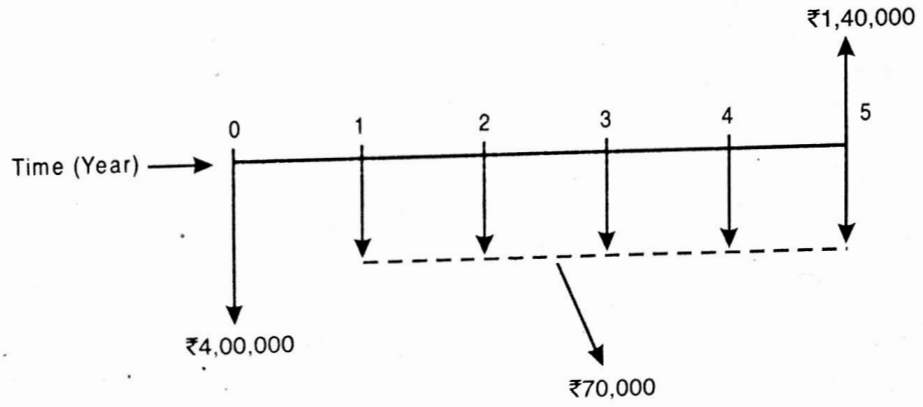

Fig. 4.14 : Cash flow diagram of Alternative-2

Now the equivalent present worth of Alternative - 2 i.e., PW_2 is calculated as follows;

The initial cost, P = ₹ 4,00,000 (cash outflow),

Annual operating and maintenance cost, A = ₹ 70,000 (cash outflow),

Salvage value, F = ₹ 1,40,000 (cash inflow).

$PW_2 = -4,00,000 - 70,000\ (P/A, i, n) + 1,40,000\ (P/F, i, n)$

$PW_2 = -4,00,000 - 70,000\ (P/A, 10\%, 5) + 1,40,000\ (P/F, 10\%, 5)$

$$PW_2 = -4,00,000 - 70,000 \times \frac{(1+i)^n - 1}{i(1+i)^n} + 1,40,000 \times \frac{1}{(1+i)^n}$$

$$PW_2 = -4,00,000 - 70,000 \times \frac{(1+0.1)^5 - 1}{0.1(1+0.1)^5} + 1,40,000 \times \frac{1}{(1+0.1)^5}$$

$$PW_2 = -4,00,000 - 70,000 \times 3.79 + 1,40,000 \times 0.62$$

$$\mathbf{PW_2 = -₹\ 5,78,500}$$

Comparing the equivalent present worth of both the alternatives, it is observed that Alternative-2 will be selected as it shows lower negative equivalent present worth compared to Alternative-1 at the interest rate of 10% per year.

Example 4.9

Two alternatives for purchase of Transit mixer having same useful life. Proposed the details of cash flow as follows.

Alternative-1 : Initial purchase cost = ₹600000, Annual operating and maintenance cost = ₹40000, Expected salvage value = ₹250000, Useful life = 5 years.

Alternative-2 : Initial purchase cost = ₹400000, Annual operating and maintenance cost = ₹70000, Expected salvage value = ₹1,40,000, Useful life = 5 years.

The annual revenue to be generated from production of concrete (by concrete mixer) from Alternative-1 and Alternative - 2 are ₹1,00,000 and ₹90000 respectively. Compute the equivalent present worth of the alternatives at the rate of interest 10% per year and find out the economical alternative.

Solution :

The cash flow diagram of Alternative-1 is shown in Fig. 4.15.

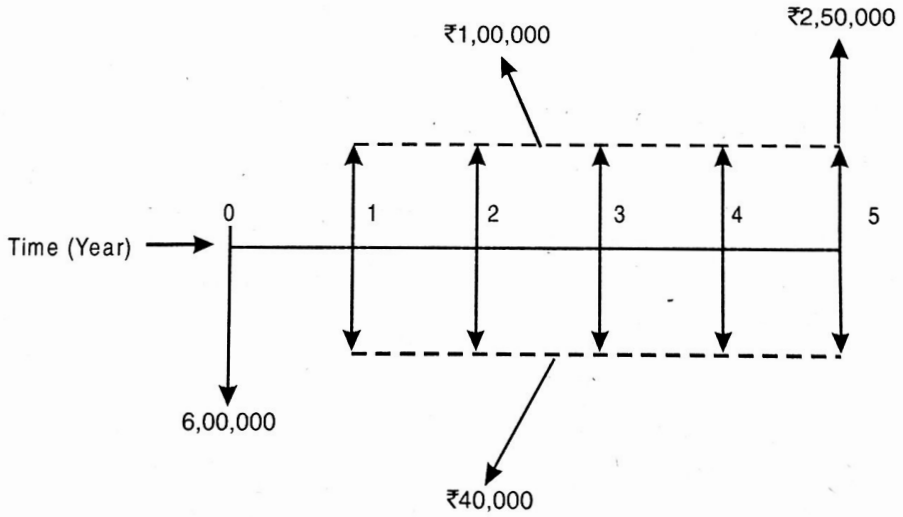

Fig. 4.15 : Cash flow diagram of Alternative-1

The equivalent present worth of Alternative-1 is calculated as follows;

$$PW_1 = -600000 - 40000\ (P/A, i, n) + 100000\ (P/A, i, n) + 250000\ (P/F, i, n)$$

$$PW_1 = -600000 - 40000\ (P/A, 10\%, 5) + 10000\ (P/A, 10\%, 5) + 250000\ (P/F, 10\%, 5)$$

$$PW_1 = -600000 + (100000 - 40000)\,(P/A, 10\%, 5) + 250000\,(P/F, 10\%, 5)$$

$$PW_1 = -600000 + 60000\,(P/A, 10\%, 5) + 250000\,(P/F, 10\%, 5)$$

$$PW_1 = -600000 + 60000 \times \frac{(1+i)^n - 1}{i(1+i)^n} + 2{,}50{,}000 \times \frac{1}{(1+i)^n}$$

$$PW_1 = -600000 + 60000 \times \frac{(1+0.1)^5 - 1}{0.1(1+0.1)^5} + 2{,}50{,}000 \times \frac{1}{(1+0.1)^5}$$

$$PW_1 = -600000 + 60000 \times 3.79 + 2{,}50{,}000 \times 0.62$$

$$\boldsymbol{PW_1 = -₹2{,}17{,}600}$$

The cash flow diagram of Alternative-2 is shown in Fig. 4.16.

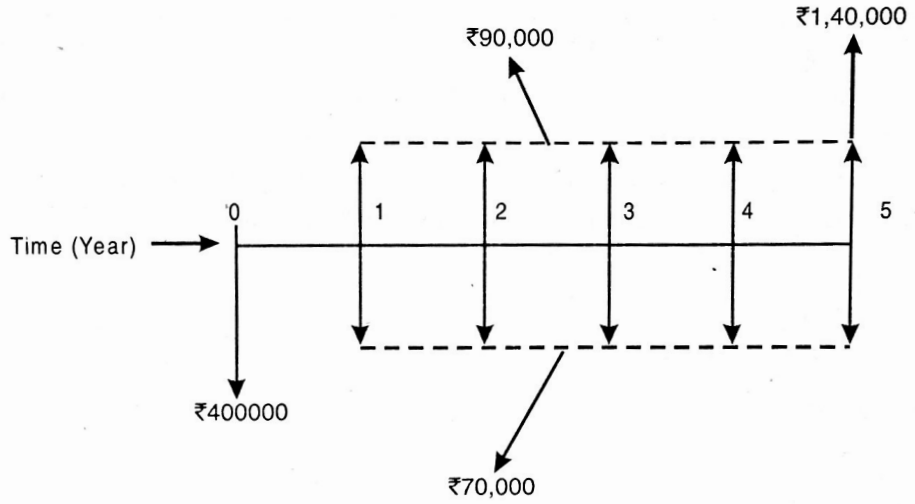

Fig. 4.16 : Cash flow diagram of Alternative-2

Now the equivalent present worth of Alternative-2 is calculated as follows;

$$PW_2 = -400000 - 70000\,(P/A, i, n) + 90000\,(P/A, i, n) + 140000\,(P/F, i, n)$$

$$PW_2 = -400000 - 70000\,(P/A, 10\%, 5) + 90000\,(P/A, 10\%, 5) + 140000\,(P/F, 10\%, 5)$$

$$PW_2 = -400000 + (90000 - 70000)\,(P/A, 10\%, 5) + 140000\,(P/F, 10\%, 5)$$

$$PW_2 = -400000 + 20000\,(P/A, 10\%, 5) + 140000\,(P/F, 10\%, 5)$$

$$PW_2 = -400000 + 20000 \times \frac{(1+i)^n - 1}{i(1+i)^n} + 140000 \times \frac{1}{(1+i)^n}$$

$$PW_2 = -400000 + 20000 \times \frac{(1+0.1)^5 - 1}{0.1(1+0.1)^5} + 140000 \times \frac{1}{(1+0.1)^5}$$

$$PW_2 = -400000 + 20000 \times 3.79 + 140000 \times 0.62$$

$$\boldsymbol{PW_1 = -₹\,2{,}37{,}400}$$

Comparing the equivalent present worth of the both the alternatives, it is observed that Alternative-1 will be selected as it shows lower cost compared to Alternative-2. The annual revenue to be generated by the alternatives made the difference as compared to the outcome obtained in Example-4.8 (Alternative 2 selected).

Example 4.10

An engineer has two bids for an excavator to be installed in a new building. The details of the bids for the excavator are as follows:

Bid	*Engineer's estimates*		
	Initial cost (₹)	*Service life (years)*	*Annual operations & maintenance cost (₹)*
Company A	**10,50,000**	**15**	**60,000**
Company B	**11,00,000**	**15**	**70,500**

Determine which bid should be accepted, based on the present worth method of comparison assuming 18% interest rate, compounded annually.

Solution :

Bid 1 : Company A

Initial cost, P = ₹ 10,50,000

Annual operation and maintenance cost, A = ₹ 60,000

Life = 15 years

Interest rate, i = 18%, compounded annually.

The cash flow diagram of bid 1 is shown in Fig. 4.17.

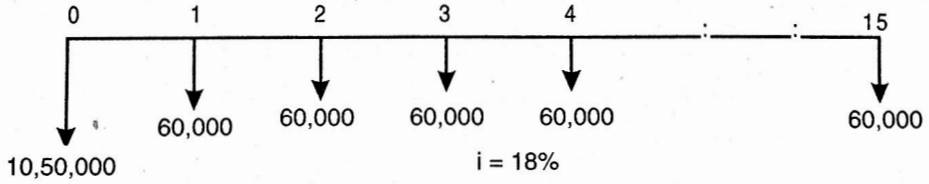

Fig. 4.17 : Cash flow diagram for bid 1

The present worth of the bid 1

$$PW(18\%) = 10,50,000 + 60,000\ (P/A, 18\%, 15)$$
$$= 10,50,000 + 60,000 \times 5.091$$
$$= ₹\ 13,55,460$$

Bid 2 : Company B

Initial cost, P = ₹ 11,00,000

Annual operation and maintenance cost, A = ₹ 70,000

Life = 15 years

Interest rate, i = 18%, compounded annually.

The cash flow diagram of bid 2 is shown in Fig. 4.18.

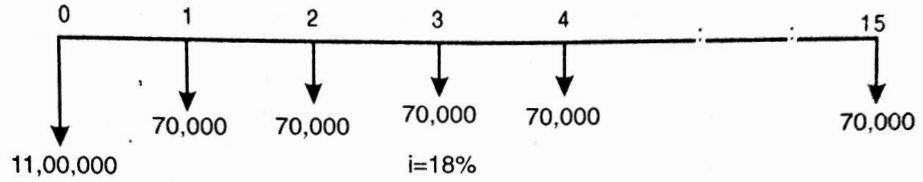

Fig. 4.18 : Cash flow diagram of bid 2

The present worth of the bid 2 :

$$PW(15\%) = 11,00,000 + 70,000\ (P/A, 18\%, 15)$$
$$= 11,00,000 + 70,000 \times 5.091$$
$$= ₹\ 14,56,370$$

The total present worth cost of bid 1 is less than that of bid 2. Hence, bid 1 is to be selected for implementation. That is, the excavater from Company A is to be purchased.

Example 4.11

A wood company is planning to buy a fully automated wood carving machine. If it is purchased under down payment, the cost of the machine is ₹20,00,000. If it is purchased under installment basis, the company has to pay 25% of the cost at the time of purchase and the remaining amount in 10 annual equal installments of ₹ 2,50,000 each. Select the best alternative for the company using the present worth basis at i = 18%, compounded annually.

Solution :

There are two alternative available for the company.

a) Down payment of ₹20,00,000

b) Down payment of $₹\frac{25}{100} \times 20,00,000$ = ₹ 5,00,000 and 10 annual equal installments of ₹ 2,50,000 each

Present worth calculation of the second alternative. The cash flow diagram of the second alternative is shown in Fig. 4.19.

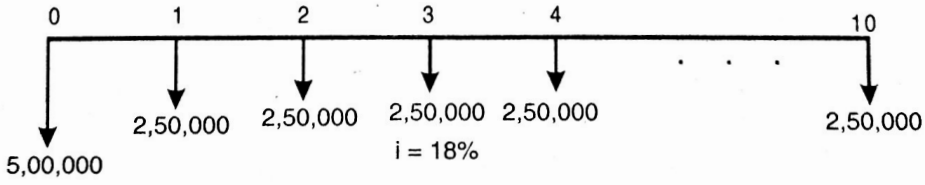

Fig. 4.19 : Cash flow diagram for the second alternative

The present worth of the above cash flow diagram is computed as

$$PW(18\%) = 5,00,000 + 2,50,000\ (P/A, 18\%, 10)$$
$$= 5,00,000 + 2,50,000 \times 4.49$$
$$= ₹16,22,500$$

The present worth of this option is ₹16,22,500, which is less than the first option of complete down payment of ₹20,00,000. Hence, the company should select the second alternative to buy the fully automated wood carving machine.

4.22.2 Comparison of alternatives by annual equivalent method

In the annual equivalent method of comparison, first the annual equivalent cost or the revenue of each alternative will be computed. Then the alternative with the maximum annual equivalent revenue in the case of revenue-based comparison or with the minimum annual equivalent cost in the case of cost based comparison will be selected as the best alternative.

Example 4.12

A company invests in one of the two mutually exclusive alternatives. The life of both alternatives in estimated to be 5 years with the following investments, annual returns and salvage values.

	Alternative	
	X	Y
Investment (₹)	– 2,50,000	– 2,75,000
Annual equal return (₹)	+ 90,000	+ 1,00,000
Salvage value (₹)	+ 20,000	+ 50,000

Determine the best alternative based on the annual equivalent method by assuming $i = 20\%$.

Solution :

Alternative X

Initial cost, $P = ₹2,50,000$

Annual equal return, $X = ₹90,000$

Salvage value at the end of machine life, $F = ₹20,000$

Life = 5 years

Interest rate, $i = 20\%$, compounded annually.

The cash flow diagram for alternative A is shown in Fig. 4.20.

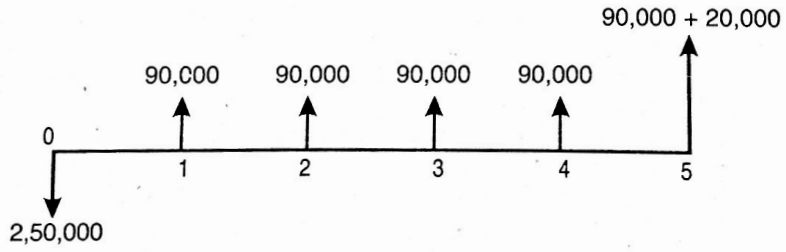

Fig. 4.20 : Cash flow diagram of alternative X

The annual equivalent revenue for alternate X

$$AE_X(20\%) = -2,50,000\ (A/P, 20\%, 5) + 90,000 + 20,000\ (A/F, 20\%, 5)$$
$$= -2,50,000(0.334) + 90,000 + 20,000(0.134)$$
$$= ₹9,180$$

Alternative B

Initial investment, P = ₹2,75,000

Annual equal return, A = ₹1,00,000

Salvage value at the end of machine life, F = ₹ 50,000

Life = 5 years

Interest rate, i = 20%, compounded annually.

The cash flow diagram for alternative B is shown in Fig. 4.20.

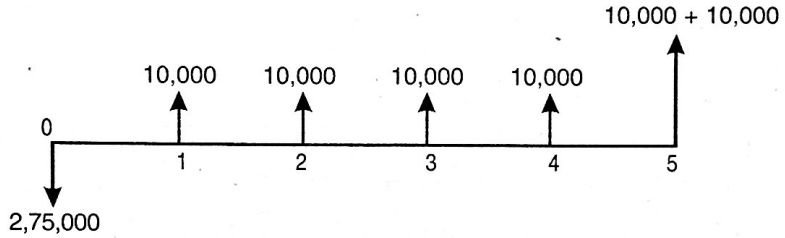

Fig. 4.21 : Cash flow diagram of alternative X

The annual equivalent revenue for alternative Y

$$AE_Y(20\%) = -2,75,000\ (A/P, 20\%, 5) + 1,00,000 + 50,000\ (A/F, 20\%, 5)$$
$$= -2,75,000(0.334) + 1,00,000 + 50,000(0.134)$$
$$= ₹14,850$$

The annual equivalent net return of alternative Y is more than that of alternative X. Thus, the company should select alternative Y.

Example 4.13 :

There are two alternative for purchasing a transit mixer and following are the cash flow details;

Alternative-1 : Initial purchase cost = ₹ 600000, Annual operating and maintenance cost = ₹20000, Expected salvage value = ₹250000, Useful life = 5 years.

Alternative-2 : Initial purchase cost = ₹400000, Annual operating and maintenance cost = ₹70000, Expected salvage value = ₹140000, Useful life = 5 years.

The annual revenue to be generated from production of concrete (by Transit mixer) from Alternative-1 and Alternative - 2 are ₹100000 and ₹90000 respectively. Compute the equivalent uniform annual worth of the alternatives at the interest rate of 10% per year and find out the economical alternative.

Solution :

The cash flow diagram of Alternative-1 i.e., Fig. 4.21 is shown here again for ready reference.

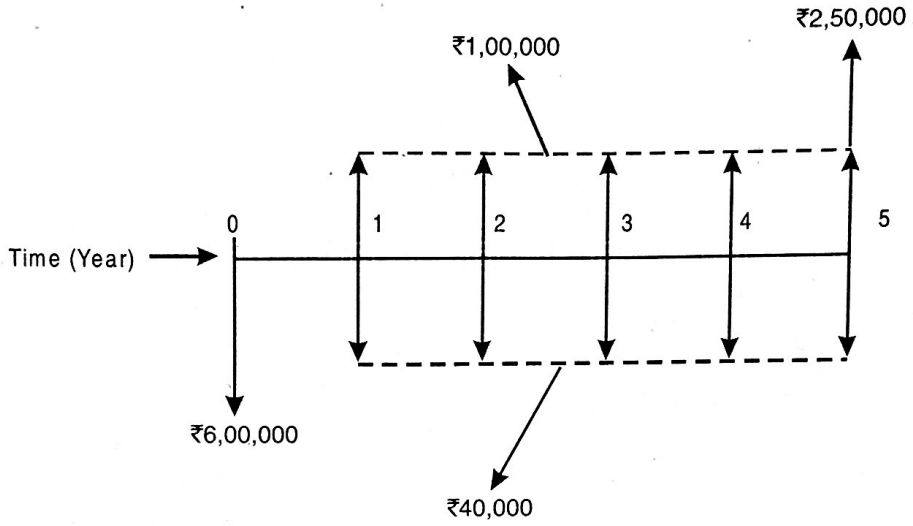

Fig. 4.21 : Cash flow diagram of Alternative-1

The equivalent uniform annual worth of Alternative-1

$AW_1 = -600000\ (A/P, i, n) - 40000 + 100000 + 250000\ (A/F, i, n)$

$AW_1 = -600000\ (A/P, 10\%, 5) - 40000 + 100000 + 250000\ (A/F, 10\%, 5)$

₹40000 and ₹10000 are annual amounts.

$AW_1 = -600000 \times 0.264 + (100000 - 40000) + 250000 \times 0.164$

$\boldsymbol{AW_1 = -₹57,400}$

The cash flow diagram of Alternative-2 is shown in Fig. 4.22.

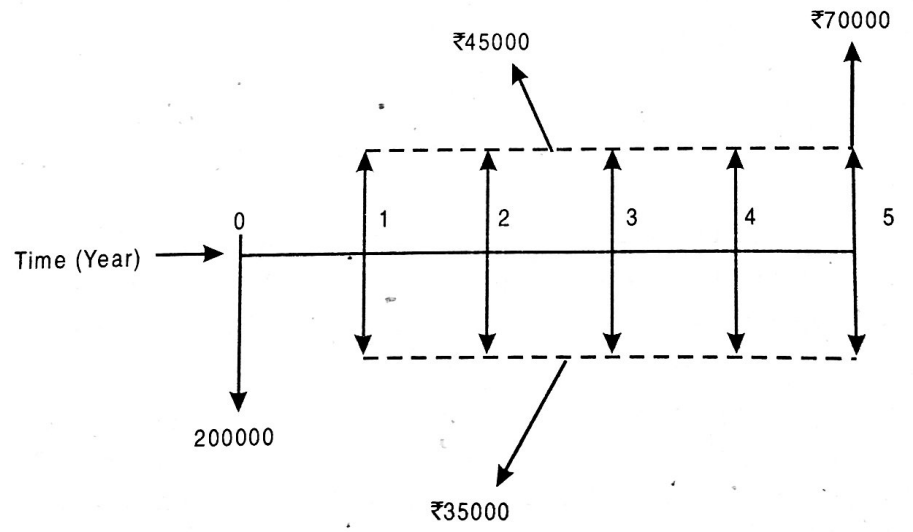

Fig. 4.22 Cash flow diagram of Alternative-2

The equivalent uniform annual worth of Alternative-2

$AW_2 = -400000\ (A/P, i, n) - 70000 + 90000 + 14000\ (A/F, i, n)$

$AW_2 = -400000\ (A/P, 10\%, 5) - 70000 + 90000 + 140000\ (A/F, 10\%, 5)$

₹70000 and ₹90000 are annual amounts.

$AW_2 = -400000 \times 0.264 + (90000 - 70000) + 14000 \times 0.164$

$\boldsymbol{AW_2 = -₹62{,}640}$

From this comparison, it is observed that Alternative-1 will be selected as it shown lower negative equivalent uniform annual worth compared to Alternative-2.

4.22.3 Comparison of alternatives by capitalized method

Capitalized cost represents the present worth of an alternative for a project that is going to serve for a longer period of time i.e., for an infinite period of time. As the name indicates, it refers to the present worth of mainly cost or expenditures (cash outflows) of the alternative over infinite period of time. Capitalized worth refers to present worth of expenditures and revenues of an alternative over infinite period of time. The capitalized cost method is used for comparison of mutually exclusive alternatives which have perpetual service life (assumed to serve forever). The examples of this kind of projects are bridges, dams, irrigation projects, water supply systems for cities, pipeline projects etc. This method an also be used for finding out the capitalized cost of permanent fellowship/scholarship endowment in educational institutes and other organizations. The present worth of a uniform amount series is given by;

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

P = present worth, A = end of year payment of uniform amount series, i = interest rate per year and n = number of interest periods.

The above equation can be rewritten as;

$$P = \frac{A}{i}\left[\frac{(1+i)^n}{(1+i)^n} - \frac{1}{(1+i)^n}\right]$$

$$P = \frac{A}{i}\left[-\frac{1}{(1+i)^n}\right]$$

In the above expression, when 'n' approaches infinity (i.e., for perpetual cash flow series), the term "$1/(1 + i)^n$" gets neglected and present worth 'P' becomes capitalized cost/worth, the expression of which is given by;

$$P = \frac{A}{i}\ ; \mathrm{A} = \mathrm{P} \times \mathrm{i}$$

The capitalized cost of a single amount occurring at regular intervals in future period of time is calculated by first finding out the equivalent uniform annual worth "A" of the uniform series and then dividing by the interest rate '*i*'.

The capitalized cost can also be used for comparison of two or more mutually exclusive alternatives which are assumed to serve perpetually. In this case the comparison of the alternatives is made over same time period i.e. infinite period of time. The alternative that shows lowest capitalized cost is selected as the best alternative.

Example 4.14 :

A concrete road project has an initial cost of ₹ 1.5 crores and annual operating and maintenance cost of ₹9 lakhs. Further the project will have one time major repair work (old bridge) of ₹25 lakhs at the end of 15 year. Find out the capitalized cost of the alternative if interest rate is 12% per year.

Solution :

The capitalized cost of the alternative is equal to sum of the initial cost, present worth of one time major repair cost and capitalized cost of the annual operating and maintenance cost.

The total capitalized cost of the alternative is given by;

Capitalized Cost = – 15000000 – 2500000 (*P*/*F*, *i*, *n*) – 900000/*i*

Capitalized Cost = – 15000000 – 2500000 (*P*/*F*, 12%, 15) – 900000/0.12

Capitalized Cost = – 15000000 – 2500000 × 0.183 – 7500000

Capitalized cost = –₹ 22957500

Example 4.15 :

The initial cost of an urban development project which is expected to serve residents of a city perpetually is ₹3 crores. The annual operating cost is ₹10 lakhs for first 10 years and ₹12 lakhs in the subsequent years (i.e. from year 11 onwards). The expected cost of renovation at the end of every 15 years is ₹20 lakhs. Find out the capitalized cost of the project at interest rate of 8% per year.

Solution:

The cash flow diagram of the project for a part of service life is shown in Fig. 4.23. The total capitalized cost of the project is equal to sum of the initial cost and capitalized cost of annual operating cost and renovation cost.

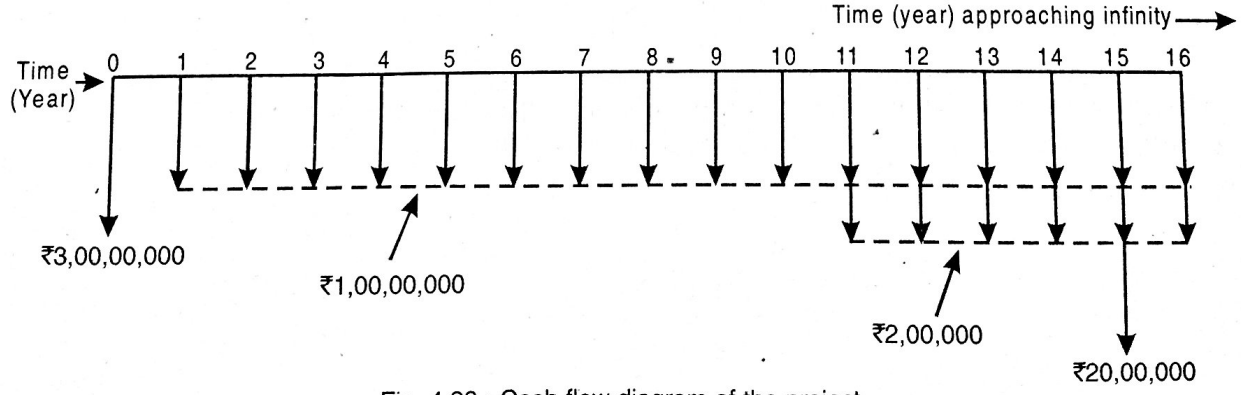

Fig. 4.23 : Cash flow diagram of the project

The annual operating cost is ₹ 1000000 for first 10 years followed by ₹1200000 thereafter. This can be represented as ₹1000000 from end of year 1 to infinite period of time and ₹200000 from end of year 11 to infinity as shown in Fig. 4.24. Thus the capitalized cost of the annual operating cost is equal to the sum of capitalized cost of these two components.

Capitalized cost of the annual operating cost:

$$\text{Capitalized Cost} = -\frac{1000000}{i} - \frac{200000}{i}(P/F,i,10)$$

In the above expression, the capitalized cost of ₹ 120000 from end of year 11 till infinity is located at the end of year 10. Now the present worth (i.e., amount at time zero) of this amount is calculated by multiplying it with single payment present worth factor.

$$\text{Capitalized Cost} = -\frac{1000000}{0.08} - \frac{200000}{0.08}(P/F, 8\%, 10)$$

$$\text{Capitalized Cost} = -12500000 - \frac{200000 \times 0.463}{0.08}$$

Capitalized Cost = –₹13657500

The capitalized cost of the annual operating cost can also be calculated by considering ₹1000000 from end of year 1 till end of year 10 and ₹1200000 from end of year 11 till infinity.

$$\text{Capitalized Cost} = -1000000(P/A,i,10) - \frac{1200000}{i}(P/F,i,10)$$

In this expression, first the present worth of uniform series with annual amount of ₹1000000 for first 10 years is calculated. Then the capitalized cost of ₹1200000 from end of year 11 till infinity is calculated.

$$\text{Capitalized Cost} = -1000000(P/A,8\%,10) - \frac{1200000}{0.08}(P/F,8\%,10)$$

$$= -1000000 \times 6.71 - \frac{1200000 \times 0.463}{0.08}$$

Capitalized Cost = – ₹13655000

The capitalized cost of annual operating cost by both ways is same.

Example 4.16 :

There are two alternatives for a sewage treatment project in a city. The details of cash flow of the alternatives are shown below.

Alternative-1

Initial cost = ₹30000000

Annual operating cost = ₹2000000

Cost of renovation = ₹5000000 at the end of every 20 years

One time upgrading cost = ₹6000000 at the end of 25 year

Alternative-2

Initial cost = ₹35000000

Annual operating cost = ₹2500000

Cost of renovation = ₹7500000 at the end of every 20 years

Compare the alternatives on the basis of capitalized cost and find out the economical alternative if the rate of interest is 9% per year

Solution :

The capitalized cost of Alternative-1 will be equal to initial cost plus the capitalized cost of annual operating cost, periodic renovation cost and one time upgrading cost.

The capitalized cost of Alternative-1

$$Capitalized\ Cost = -30000000 - \frac{2000000}{i} - \frac{5000000(A/F,i,20)}{i} - 6000000(P/F,i,25)$$

$$Capitalized\ Cost = -30000000 - \frac{2000000}{0.09} - \frac{5000000(A/F,9\%,20)}{0.09} - 6000000(P/F,9\%,25)$$

$$= -30000000 - 22222222 - \frac{5000000 \times 0.0195}{0.09} - 6000000 \times 0.116$$

$$= -30000000 - 22222222 - 108333.33 - 780640$$

Capitalized cost = – ₹ 5,33,75,155

The capitalized cost of Alternative - 2 will be equal to initial cost plus the capitalized cost of annual operating cost and periodic renovation cost.

The capitalized cost of Alternative - 2

$$Capitalized\ Cost = -35000000 - \frac{2500000}{i} - \frac{7500000(A/F,i,20)}{i}$$

$$= -35000000 - \frac{2500000}{0.09} - \frac{7500000(A/F,9\%,20)}{0.09}$$

$$= -35000000 - 27777777 - \frac{7500000 \times 0.0195}{0.09}$$

Capitalized cost = – ₹ 6,44,02,777

The capitalized cost of alternative-1 and Alternative-2 are found to be **₹ 53375155** and **₹ 64402777** respectively. Thus Alternative-1 is the economical as it shows lower capitalized cost (lower negative value) as compared to Alternative-2.

4.22.4 Comparison of alternatives by rate of return method

The rate of return of a cash flow pattern is the interest rate at which the present worth of that cash flow pattern reduces to zero. In this method of comparison, the rate of return for each alternative

is computed. Then the alternative which has the highest rate of return is selected as the best alternative. In this type of analysis, the expenditures are always assigned with a negative sign and the revenues/inflows are assigned with a positive sign. A generalized cash flow diagram to demonstrate the rate of return method of comparison is presented in Fig. 4.24.

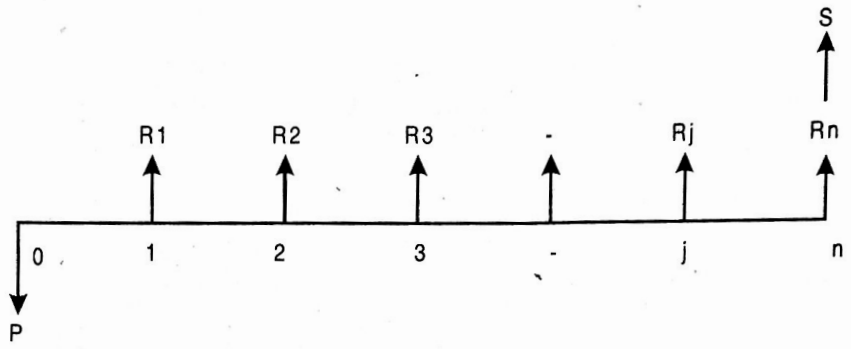

Fig. 4.24 : Generalized cash flow diagram

In the above cash flow diagram, P represents an initial investment, Rj the net revenue at the end of the jth year, and S the salvage value at the end of the nth year.

The first step is to find the net present worth of the cash flow diagram using the following expression at a given interest rate, i.

$$PW(i) = -P + R1/(1+i)^1 + R2/(1+i)^2 + ... + Rj/(1+i)^j + ... + Rn/(1+i)^n + S/(1+i)^n$$

Now, the above function is to be evaluated for different values of i until the present worth function reduces to zero, as shown in Fig. 4.25. In the figure 4.25, the present worth goes on decreasing when the interest rate is increased. The value of i at which the present worth curve cuts the X-axis is the rate of return of the given proposal/project. It will be very difficult to find the exact value of i at which the present worth function reduces to zero.

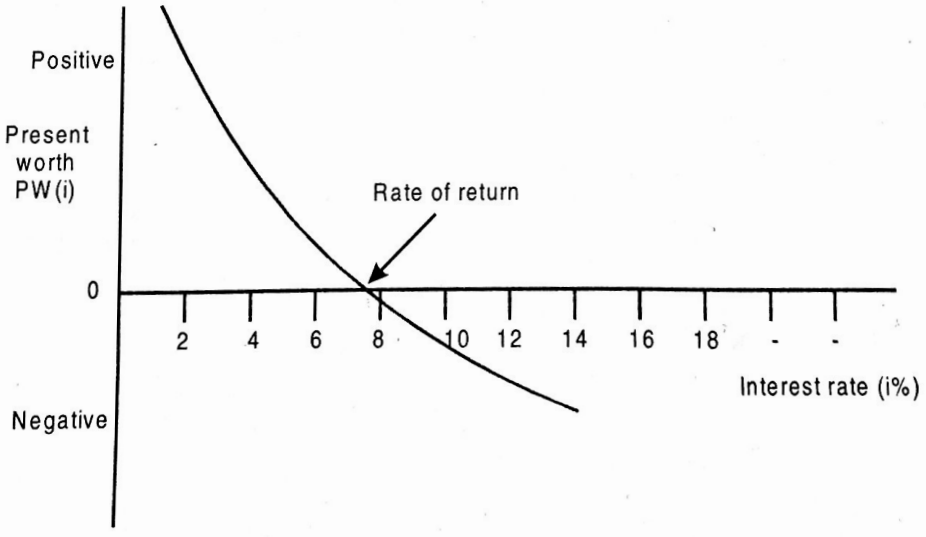

Fig. 4.25 : Present worth function graph

So, one has to start with an intuitive value of i and check whether the present worth function is positive. If so, increase the value of i until $PW(i)$ becomes negative. Then, the rate of return is determined by interpolation method in the range of values of i for which the sign of the present worth function changes from positive to negative.

Example 4.17 :

A firm has identified three mutually exclusive investment proposals for new project whose details are given below. The life of all the three alternatives is estimated to be five years with negligible salvage value. The minimum attractive rate of return for the firm is 12%.

	Alternative		
	A1	**A2**	**A3**
Investment	**₹1,50,000**	**₹3,10,000**	**₹3,25,000**
Annual net income	**₹50,000**	**₹90,000**	**₹91,000**

Find the best alternative based on the rate of return method of comparison.

Solution :

Calculation of rate of return for alternative A1

Initial outlay = ₹1,50,000

Annual profit = ₹50,000

Life = 5 years

The cash flow diagram for alternative A1 is shown in Fig. 4.26.

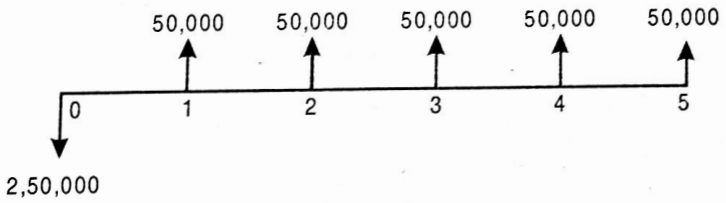

Fig. 4.26 Cash flow diagram for alternative A1.

The formula for the net present worth of alternative A1 is given as

$$PW(i) = -1,50,000 + 50,000(P/A, i, 5)$$

When $i = 10\%$

$$PW(10\%) = -1,50,000 + 50,000(P/A, 10\%, 5)$$
$$= -1,50,000 + 50,000(3.79)$$
$$= ₹\ 39,500$$

When $i = 12\%$

$$PW(12\%) = -1,50,000 + 50,000(P/A, 12\%, 5)$$
$$= -1,50,000 + 50,000(3.60)$$
$$= ₹\ 30,000$$

When $i = 18\%$

$$PW(18\%) = -1{,}50{,}000 + 50{,}000(\text{P/A}, 18\%, 5)$$
$$= -1{,}50{,}000 + 50{,}000(3.12)$$
$$= ₹\,6{,}000$$

When $i = 21\%$

$$PW(21\%) = -1{,}50{,}000 + 50{,}000(\text{P/A}, 21\%, 5)$$
$$= -1{,}50{,}000 + 50{,}000(2.92)$$
$$= ₹\,4{,}000$$

Therefore, the rate of return of the alternative A1 is

$$i = 18\% + \frac{6{,}000 - 0}{6{,}000 - (-4{,}000)} \times (3\%)$$
$$= 15\% + 1.8\%$$
$$= 16.80\%$$

Calculation of rate of return for alternative A2

Initial outlay = ₹ 2,80,000

Annual profit = ₹ 90,000

Life of alternative A2 = 5 years

The cash flow diagram for alternative A2 is shown in Fig. 4.27.

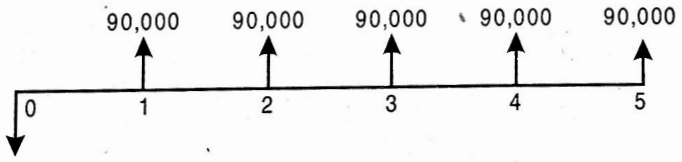

Fig. 4.27 Cash flow diagram for alternative A2.

The formula for the net present worth of this alternative is

$$PW(i) = -3{,}10{,}000 + 90{,}000(\text{P/A}, i, 5)$$

When $i = 12\%$,

$$PW(12\%) = -3{,}10{,}000 + 90{,}000(\text{P/A}, 12\%, 5)$$
$$= -3{,}10{,}000 + 90{,}000(3.60)$$
$$= ₹14{,}000$$

When $i = 15\%$,

$$PW(15\%) = -3{,}10{,}000 + 90{,}000(\text{P/A}, 13\%, 5)$$
$$= -3{,}10{,}000 + 90{,}000(3.35)$$
$$= ₹\,8500$$

Therefore, the rate of return of alternative A2 is

$$i = 12\% + \frac{14000 - 0}{14000 - (-8500)} \times (3\%)$$

$$= 12\% + 1.86\%$$

$$= 13.86\%$$

Calculation of rate of return for alternative A3

Initial outlay = ₹ 325,000

Annual profit = ₹ 91,000

Life of alternative A3 = 5 years

The cash flow diagram for alternative *A*3 is depicted in Fig. 4.28.

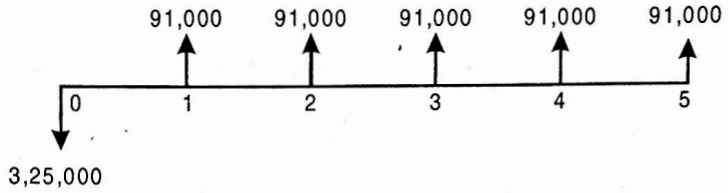

Fig. 4.28 Cash flow diagram for alternative A3.

The formula for the net present worth of this alternative *A*3 is

$$PW(i) = -3,25,000 + 91,000(\text{P/A}, i, 5)$$

When $i = 12\%$,

$$PW(12\%) = -3,25,000 + 99,000(\text{P/A}, 11\%, 5)$$

$$= -3,25,000 + 91,000(3.6)$$

$$= ₹\,2600$$

When $i = 11\%$,

$$PW(11\%) = -3,25,000 + 91,000(\text{P/A}, 11\%, 5)$$

$$= -3,25,000 + 91,000(3.69)$$

$$= ₹\,-10790$$

Therefore, the rate of return for alternative A3 is

$$i = 11\% + \frac{10790 - 0}{10790 - (-2600)} \times 1\%$$

$$= 11.80\%$$

The rates of return for the three alternatives are now tabulated.

Alternative	A1	A2	A3
Rate of return	16.80%	13.86%	11.80%

From the above data, it is clear that the rate of return for alternative A3 is less than the minimum attractive rate of return of 11.8%. So, it should not be considered for comparison. the remaining two alternatives are qualified for consideration. Among the alternatives A1 and A2, the rate of return of alternative A1 is greater than that of alternative A2. Hence, alternative A1 should be selected.

Example 4.18 :

A company is planning to small scale business activity. It has two alternatives and the corresponding cash flows are tabulated below. Each alternative has a life of five years and a negligible salvage value. The minimum attractive rate of return for the company is 12%. Suggest the best alternative to the company.

	Initial investment (₹)	*Yearly revenue* (₹)
Alternative 1	**9,50,000**	**3,50,000**
Alternative 2	**16,00,000**	**5,50,000**

Solution :

Alternative **1**

Initial outlay = Rs. 10,00,000

Annual profit = Rs. 3,50,000

Life of alternative 1 = 5 years

The cash flow diagram for alternative 1 is illustrated in Fig. 4.29.

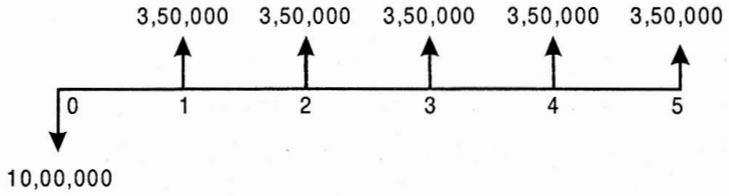

Fig. 4.29 Cash flow diagram for alternative 1.

The formula for the net present worth of this alternative is

$$PW_1(i) = -9,50,000 + 3,50,000(\text{P/A}, i, 5)$$

$$PW_1(15\%) = -9,50,000 + 3,50,000(\text{P/A}, 15\%, 5)$$

$$= -9,50,000 + 3,50,000(3.35)$$

$$= ₹\ 2,22,500$$

$$PW_1(17\%) = -9,50,000 + 3,50,000(\text{P/A}, 17\%, 5)$$

$$= -9,50,000 + 3,50,000(3.199)$$

$$= ₹\ 1,69,650$$

$$PW_1(20\%) = -9,50,000 + 3,50,000(P/A, 20\%, 5)$$
$$= -9,50,000 + 3,50,000(2.99)$$
$$= ₹\ 96,500$$
$$PW_1(25\%) = -9,50,000 + 3,50,000(P/A, 25\%, 5)$$
$$= -9,50,000 + 3,50,000(2.69)$$
$$= ₹\ -8,500$$

Therefore, the rate of return of alternative 1 is

$$i = 20\% + \frac{96,500 - 0}{96,500 - (-8,500)} \times 5\%$$
$$= 24.59\%$$

Alternative 2

Initial outlay = ₹ 16,00,000

Annual revenue = ₹ 5,50,000

Life = 5 years

The cash flow diagram for alternative 2 is depicted in Fig. 4.30.

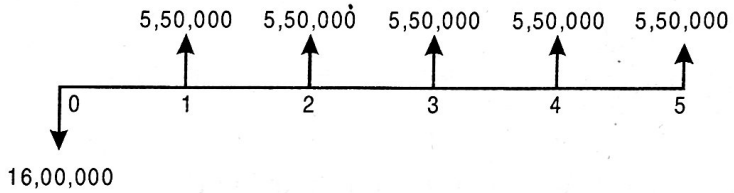

Fig. 4.30 Cash flow diagram for alternative 2.

The formula for the net present worth of this alternative 2 is :

$$PW_2(i) = -16,00,000 + 5,50,000(P/A, i, 5)$$
$$PW_2(20\%) = -16,00,000 + 5,50,000(P/A, 20\%, 5)$$
$$= -16,00,000 + 5,50,000(2.99)$$
$$= ₹\ 44,500$$
$$PW_2(22\%) = -16,00,000 + 5,50,000(P/A, 22\%, 5)$$
$$= -16,00,000 + 5,50,000(2.86)$$
$$= ₹\ -27,000$$

Thus, the rate of return of alternative 2 is

$$i = 20\% + \frac{44,500 - 0}{44,500 - (-27,000)} \times 2\%$$
$$= 21.24\%$$

Since the rate of return of alternative 1 is greater than that of the alternative 2, select alternative 1.

4.23 BREAK EVEN ANALYSIS

Break even analysis examines the relationship between the total revenue, total costs and total profits of the firm at various levels of output. It is used to determine the sales volume required for the firm to break even and the total profits and losses at other sales level. Break even analysis is a method, as said by Dominick Salnatore, of revenue and total cost functions of the firm. According to Martz, Curry and Frank, a break even analysis indicates at what level cost and revenue are in equilibrium.

In case of break even analysis, the break even point is of particular importance. Break even point is that volume of sales where the firm breaks even i.e., the total costs equal total revenue. It is, therefore, a point where losses cease to occur while profits have not yet begun. That is, it is the point of zero profit.

$$\text{BEP} = \frac{\text{Fixed Costs}}{\text{Selling price} - \text{Variable costs per unit}}$$

$$\text{For Example, } = \frac{\text{Fixed Costs Rs. 10,000}}{\text{Selling price Rs. 5 per unit} - \text{Variable costs Rs. 3 per unit}}$$

$$\text{Therefore, BEP} = \frac{₹10,000}{5-3} = 5{,}000 \text{ units}$$

The conclusion that can be drawn from the above example is that sales volume of 5000 units will be the accurate point at which the manufacturing unit would not make any loss or profit.

Uses

Break even analysis is a very generalised approach for dealing with a wide variety of questions associated with profit planning and forecasting. Some of the important practical applications of break even analysis are

- What happens to overall profitability when a new product is introduced?
- What level of sales is needed to cover all costs and earn. sav. ₹ 1,00,000 profit or a 12% rate of return?
- What happens to revenues and costs if the price of one of a company's product is hanged?
- What happens to overall profitability if a company purchases new capital equipment or incurs higher or lower fixed or variable costs?
- Between two alternative investments, which one offers the greater margin of profit (safety)?
- What are the revenue and cost implications of changing the process of production?
- Should one make, buy or lease capital equipment?

Assumptions

The break even analysis is based on certain assumptions, namely

- All costs are either perfectly variable or absolutely fixed over the entire period of production but this assumption does not hold good in practice.

- The volume of production and the volume of sales are equal; but in reality they differ.
- All revenue is perfectly variable with the physical volume of production and this assumption is not valid.
- The assumption of stable product mix is unrealistic.

The Break even Chart

The difference between price and average variable cost (P-AVC) is defined as 'profit contribution'. That is, revenue on the sale of a unit of output after variable costs are covered represents a contribution toward profit. At low rates of output, the firm may be losing money because fixed costs have not yet been covered by the profit contribution. Thus, at these low rates of output, profit contribution is used to cover fixed costs. After fixed costs are covered, the firm will be earning a profit.

A manager may want to know the output rate necessary to cover all fixed costs and to earn a "required" profit of R. Assume that both price and variable cost per unit of output (APC) are constant. Profit is equal to total revenue (P.Q.) less the sum of total variable costs (Q.QVC) and fixed costs. Thus

Profit = PQ – [(Q.AVC) + FC]

Profit = TR – TC

The break even chart shows the extent of profit or loss to the firm at different levels of activity. A break even chart may be defined as an analysis in graphic form of the relationship of production and sales to profit. The Break even analysis utilises a break even chart in which the total revenue (TR) and the total cost (TC) curves are represented by straight lines, as in Figure 4.31.

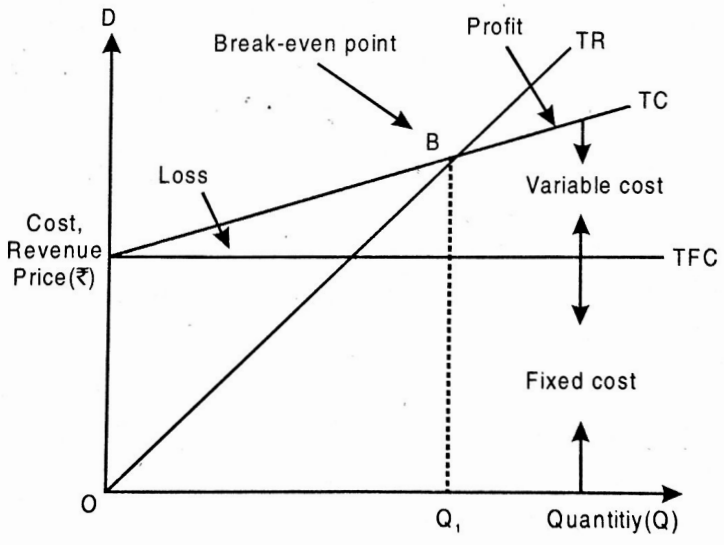

Figure : 4.31

In the figure 4.31 total revenues and total costs are plotted on the vertical axis whereas output or sales per time period are plotted on the horizontal axis. The slope of the TR curve refers to the

constant price at which the firm can sell its output. The TC curve indicates total fixed costs (TFC) (The vertical intercept) and a constant average variable cost (the slope of the TC curve). This is often the case for many firms for small changes in output or sales. The firm breaks even (with TR = TC) at Q_1 (point B in the figure 4.31) and incurs losses at smaller outputs while earnings profits at higher levels of output.

Both the total cost (TC) and total revenue (TR) curves are shown as linear. TR curve is linear as it is assumed that the price is given, irrespective of the output level. Linearity of TC curve results from the assumption of constant variable costs.

If the assumptions of constant price and average variable cost are relaxed, break even analysis can still be applied, although the key relationship (total revenue and total cost) will not be linear functions of output. Nonlinear total revenue and cost functions are shown in Figure 4.32. The cost function is conventional in the sense that at first costs increase but less than in proportion to output and then increase more than in proportion to output. There are two break even points - L and M. Note that profit which is the vertical distance between the total revenue and total cost functions, is maximised at output rate Q*.

Of the two break even points, only the first, corresponding to output rate Q_1 is relevant. When a firm begins production, management usually expects to incure losses. But it is important to know at what output rate the firm will go from a loss to a profit situation. In Figure 4.32 the firm would want to get to the break even output rate Q_1 as soon as possible and then of course, move to the profit maximising rate Q*. However, the firm would not expand production beyond Q* because this would result in a reduction of profit.

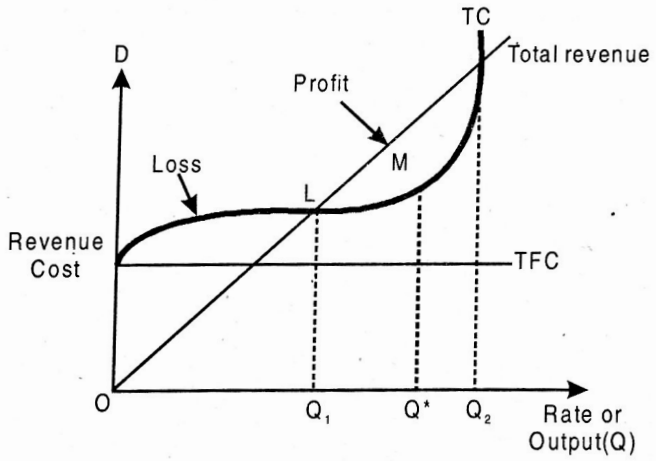

Figure : 4.32

REVIEW QUESTIONS

1. Discuss concept of engineering economy.
2. Explain Principles of engineering economics.
3. Differentiate between Micro and macro analysis.
4. Explain problem solving and decision making in engineering economics.
5. Define Interest and time value of money.
6. What is simple and compound interest? Write interest formula for both.
7. Explain briefly with interest formula for different types of
8. i) Single payment ii) Equal payment iii) Uniform gradient series
9. Define Nominal and effective interest rates.
10. Define Deferred annuities and capitalized cost.
11. Discus with suitable example comparison of alternatives for i) Present worth method ii) Annual equivalent method iii) capitalized and rate of return methods Explain Break even analysis.