MODULE-2

Moment Distribution Method

(*) Procedure : 1) FEM
2) Distribution factor (DF)
3) Moment Distribution Table
4) Diagrams (SFD, BMD & EC)

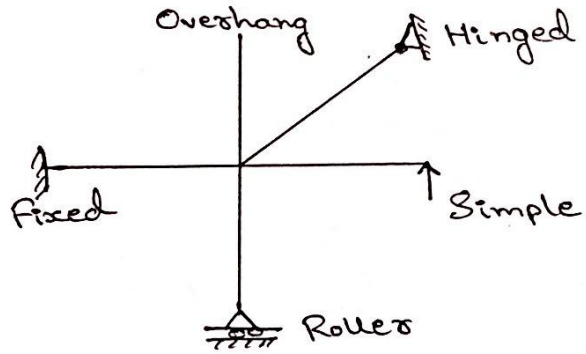
1) FEM :
Refer Unit-2 — Slope Deflection Method.

2) Distribution factor (DF) :
(For intermediate Support joints)

⇒

Joint	Member	Relative Stiffness (k)	Σk	$DF = \frac{k}{\Sigma k}$

⇒ Relative Stiffness (k)



a) For Fixed (or) Continuous support → $(k = \frac{I}{L})$

b) For Simple, Roller (or) Hinge → $(k = \frac{3}{4} \times \frac{I}{L})$

c) For Overhang → $(k = 0)$

3> Moment Distribution Table:

⇒ (a) If the far end is fixed (or) Continuous carry 50% of moment with same sign.

(b) If the far end is "Not Continuous", then there is no transfer of moments.

⇒ M.D. Table:

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF						
FEM						
Bal C.O						
Bal C.O						
Bal C.O						
Bal C.O						
Final Moments						

4> Diagrams (SFD, BMD & EC):

Refer Unit-2 —— S.D. Notes

(*) Sinking and Rotation of Support

① FEM:

(a) Additional Moment due to rotation

@ near end $= \frac{4EI\theta}{l}$

@ Far end $= \frac{2EI\theta}{l}$

(b) Additional moment due to Sinking.

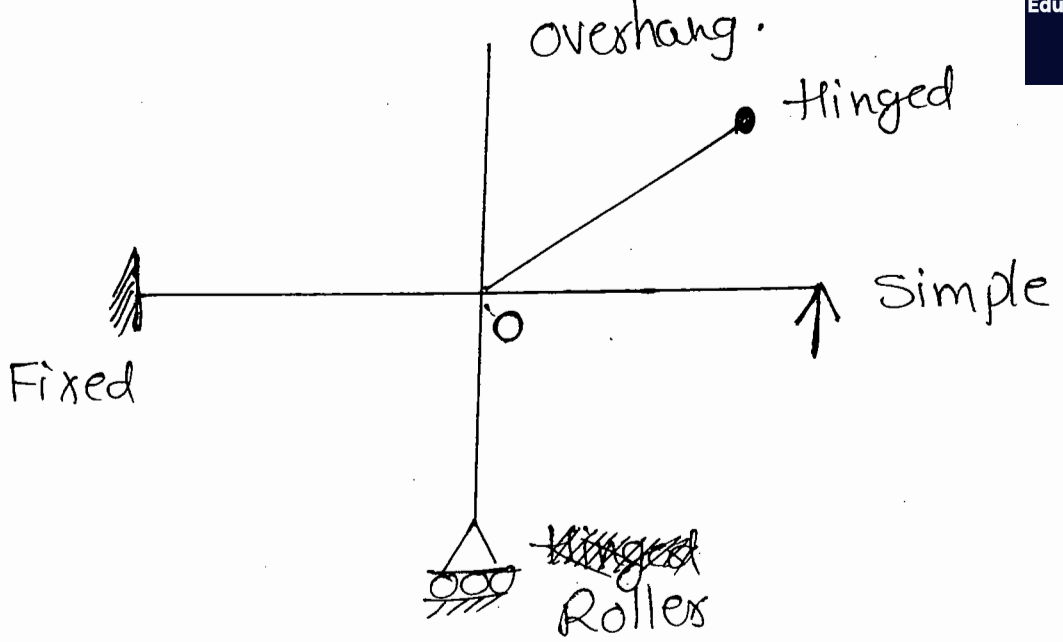
" $-\frac{6EI\delta}{l^2}$ "

Date 21/9/18

(III) Moment Distribution Method

Relative stiffness = $k = \frac{I}{l}$

"The ratio of M.I to the span of beam is called relative stiffness."

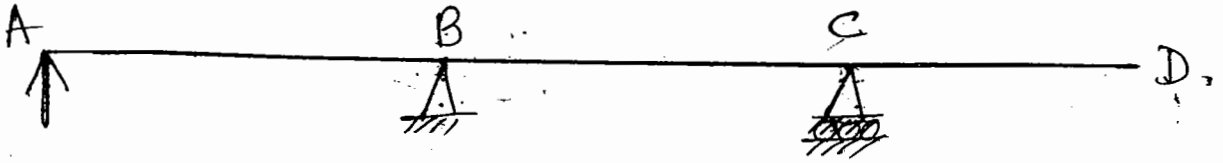


(a) For "Fixed end" or "Continuous" support } $k = \frac{I}{l}$

(b) For "Simple" or "Hinge" or "Roller" support } $k = \frac{3}{4}\frac{I}{l}$

(c) For "Overhang" → $k = 0$

Continuous support :



(i) w.r.to "B" → 'A' is Not continuous $k = \frac{3}{4}\frac{I}{l}$

'C' is Not Continuous $k = \frac{3}{4}\frac{I}{l}$.

(ii) w.r.to 'C' → 'B' is Continuous $k = I/l$

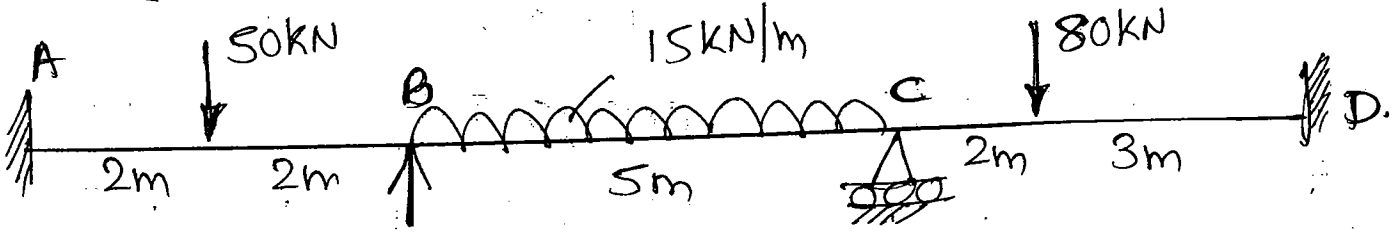
'D' is overhang $k = 0$

Carry over of Moments :-

(i) If the far end is "fixed" or "Continuous" take or carry 50% of moment with Same sign.

(ii) If far end is Not continuous, then there is no Transfer of moment.

Eg:-1] Analyse the continuous beam shown by MD method. Draw SFD, BMD & EC.



Sol:- (a) FEM

$M_{FAB} = -\frac{WL}{8} = -25\ kN\text{-}m$, $M_{FBA} = +25\ kN\text{-}m$

$M_{FBC} = -\frac{wl^2}{8} = -31.25$, $M_{FCB} = +\frac{wl^2}{12} = 31.25$

$M_{FCD} = -\frac{Wab^2}{l^2} = -57.6$, $M_{FDC} = +\frac{Wa^2b}{l^2} = 38.4$

(b) Distribution Factor (For Intermediate support)

Joint	Member	Relative Stiffness = K	Sum Σk	DF = $\frac{k}{\Sigma k}$
B	BA	$(I/l) = I/4 = 0.25I$	0.45I	$\frac{0.25}{0.45} = 0.56$
	BC	$(I/l) = I/5 = 0.20I$		$\frac{0.2}{0.45} = 0.44$
C	CB	$(I/l) = I/5 = 0.20I$	0.4I	0.5
	CD	$(I/l) = I/5 = 0.2I$		0.5

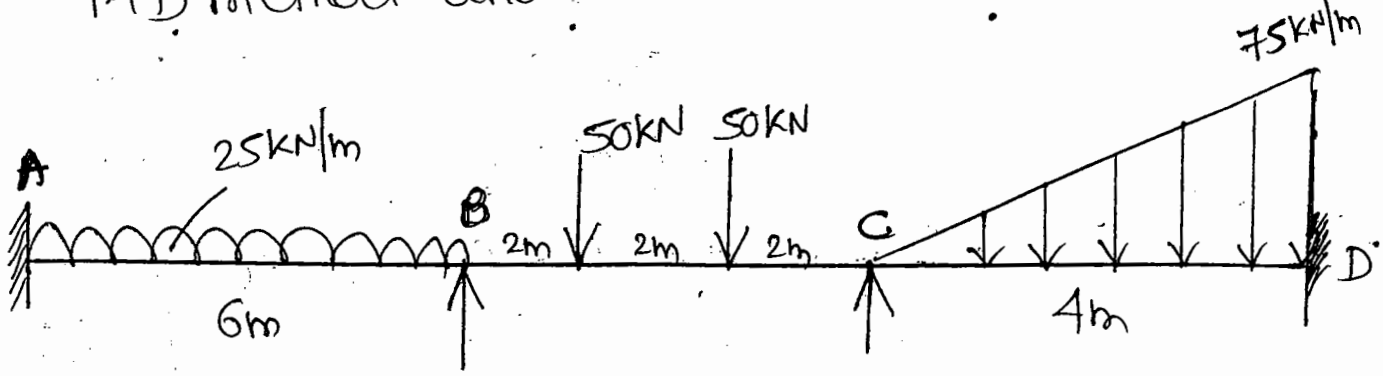
(c) Moment Distribution Table

A (Fixed) B ✓ C ✓ D (Fixed)

AB	BA	BC	CB	CD	DC	Member
-	0.56	0.44	0.5	0.5		DF
-25	25	-31.25	31.25	-57.6	38.4	FEM
	3.5	2.75	13.18	13.18		Balance
1.75		6.59	1.37		6.59	Carry over
	-3.69	-2.90	-0.68	-0.68		Bal
-1.84		-0.34	-1.45		-0.34	C.O
	0.19	0.15	0.73	0.73		Bal
0.09		0.36	0.075		0.36	C.O
	-0.20	-0.16	-0.037	-0.037		Bal
-0.10		-0.018	-0.08		-0.018	C.O
	0.01	0.008	0.04	0.04		Bal
25.10	24.81	-24.81	44.40	-44.40	44.992	Final Moments

Draw SFD, BMD and EC.

Eg:-2] Analyse the beam shown by MD method and draw BMD, SFD & EC.



Sol:2

(a) FEM

$$M_{FAB} = -\frac{wl^2}{12} = -75, \quad M_{FBA} = +75$$

$$M_{FBC} = -\frac{Wab^2}{l^2} = -\left[\frac{50\times2\times4^2}{6^2} + \frac{50\times4\times2^2}{6^2}\right] = -66.67$$

$$M_{FCB} = +\frac{Wa^2b}{l^2} = +66.67$$

$$M_{FCD} = -\frac{wl^2}{30} = \frac{-75\times4^2}{30} = -40$$

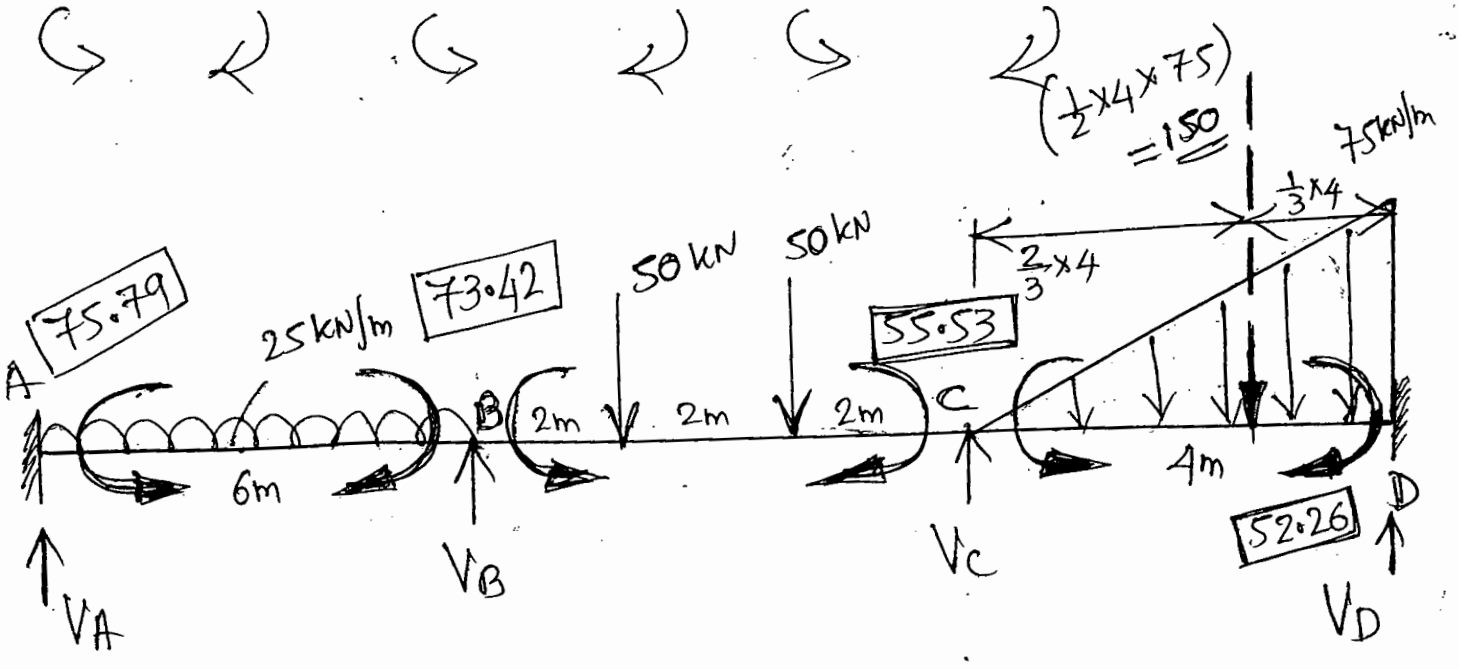
$$M_{FDC} = +\frac{wl^2}{20} = +60$$

(b) D.F : (For Intermediate Support)

	Member	Relative stiffness = K	ΣK	$DF = \frac{K}{\Sigma K}$
B	BA	$I/l = I/6 = 0.167I$	0.333I	0.5
	BC	$I/l = I/6 = 0.167I$		0.5
C	CB	$I/l = I/6 = 0.167I$	0.417I	0.4
	CD	$I/l = I/4 = 0.25I$		0.6

(C) M.D. Table

A (Fixed)		B ✓		C ✓		D (Fixed)	
AB	BA	BC	CB	CD	DC		Member
	0.5	0.5	0.40	0.60			DF
-75	75	-66.67	+66.67	-40	60		FEM
	-4.17	-4.17	-10.67	-16.0			Balance
-2.09		-5.34	-2.09		-8		C.O
	2.67	2.67	0.84	1.25			Bal
1.34		0.42	1.34		0.63		C.O
	-0.21	-0.21	-0.54	-0.80			Bal
-0.11		-0.27	-0.11		-0.4		C.O
	0.14	0.14	+0.05	+0.06			Bal
0.07		0.02	0.07		0.03		C.O
	-0.01	-0.01	-0.03	-0.04			Bal
-75.79	73.42	-73.42	55.53	-55.53	52.26		Final Values.



Reactions

$\Sigma V = 0$, $V_A + V_B + V_C + V_D = 25\times6 + 2\times50 + 150$

$= 400$ —(i)

$\Sigma M_B = 0$ (LHS)

$V_A \times 6 - 25\times6\times6/2 - 75.79 + 73.42 = 0$ $\boxed{V_A = 75.4}$

$\Sigma M_C = 0$ (RHS)

$-V_D \times 4 + \left(150 \times \frac{2}{3} \times 4\right) - 55.53 + 52.26 = 0$

$\boxed{V_D = 99.18}$

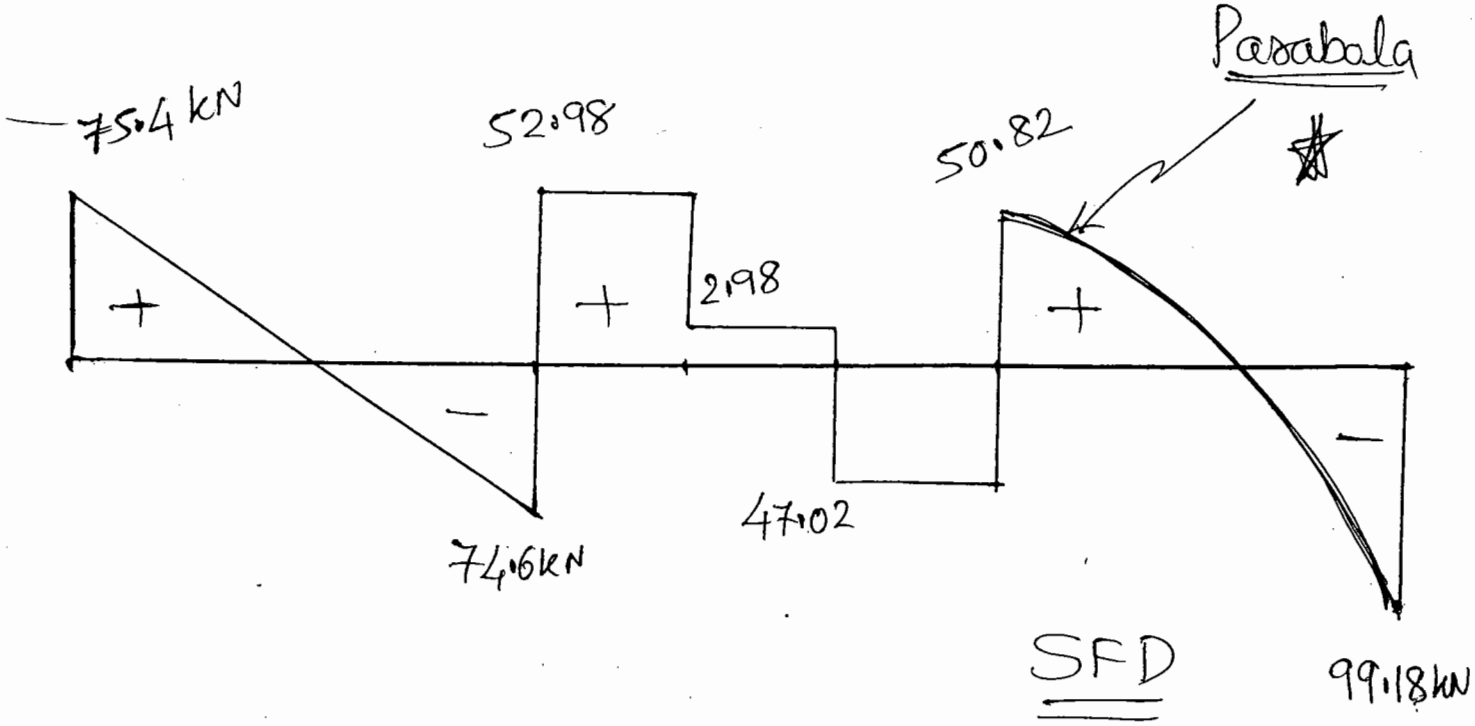
$\Sigma M_C = 0$ (LHS)

$75.4\times12 + V_B\times6 - 25\times6\times9 - 50\times2 - 50\times4$

$- 75.79 + 73.42 - 73.42 + 55.53 = 0$

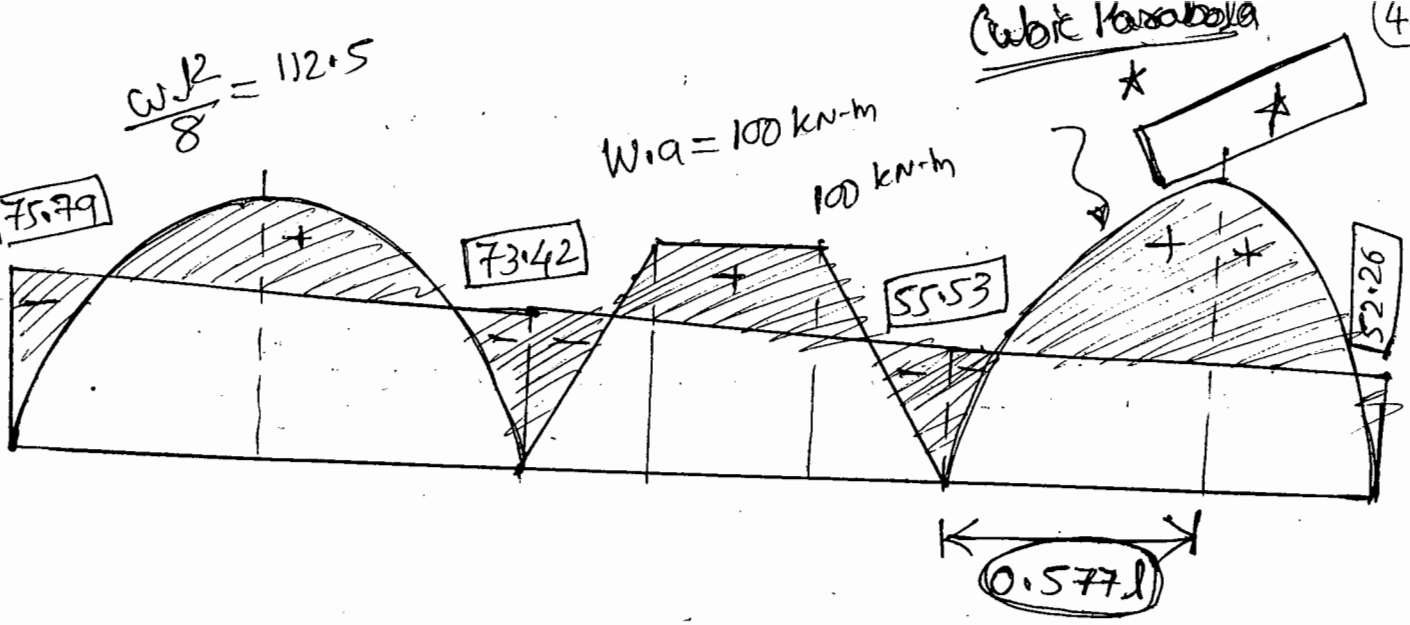
$\boxed{V_B = 127.58}$

From (i) $\boxed{V_C = 97.84}$



SFD

Cubic Parabola



$= 0.577 \times 4$

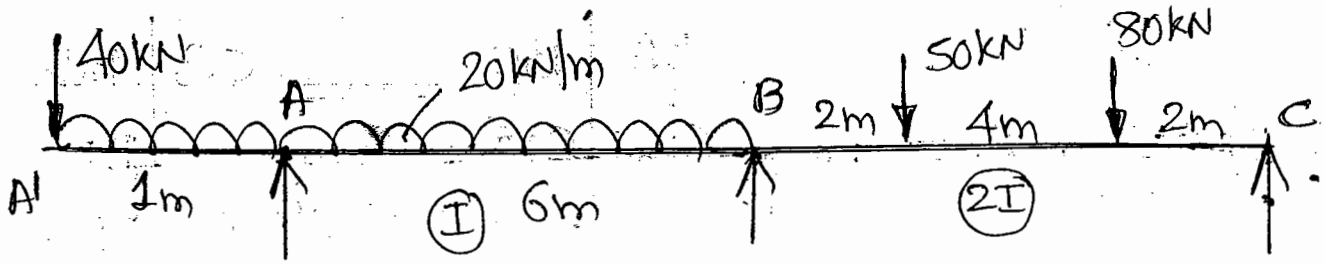
$= 2.308$ m

$M = 0.06415 \omega l^2$ ★

$= 0.06415 \times 75 \times (4)^2 =$ 77 kN-m

—x—

Eg:-3] Analyse the beam shown by M.D. method. Draw SFD, BMD & EC.

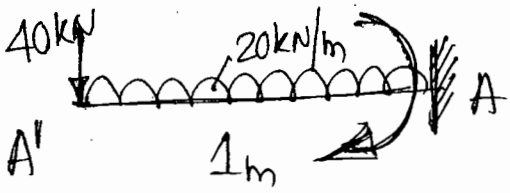


(a) FEM

$$M_{FAB} = -\frac{wl^2}{12} = -60 \text{ kN-m}, \quad M_{FBA} = +60 \text{ kN-m}$$

$$M_{FBC} = -\frac{Wab^2}{l^2} = -\left[\frac{50\times2\times6^2}{8^2} + \frac{80\times6\times2^2}{8^2}\right] = -86.25 \text{ kN-m}$$

$$M_{FCB} = +\frac{Wa^2b}{l^2} = +\left[\frac{50\times2^2\times6}{8^2} + \frac{80\times6^2\times2}{8^2}\right] = +108.75 \text{ kN-m}$$



$$M_{AA'} = +40\times1 + 20\times1\times\tfrac{1}{2} = \boxed{+50} \text{ kN-m}.$$

(b) D.F. (For Intermediate)

	Member	K	ΣK	$DF = \frac{K}{\Sigma K}$
A	AA'	0 (∵ Overhang)	0.167I	0
	AB	$\frac{I}{l} = \frac{I}{6} = 0.167I$		1
B	BA	$\frac{3}{4}\left(\frac{I}{l}\right) = \frac{3}{4}\left(\frac{I}{6}\right) = 0.125I$	0.3125I	0.40
	BC	$\frac{3}{4}\left(\frac{I}{l}\right) = \frac{3}{4}\left(\frac{2I}{8}\right) = 0.1875I$		0.60

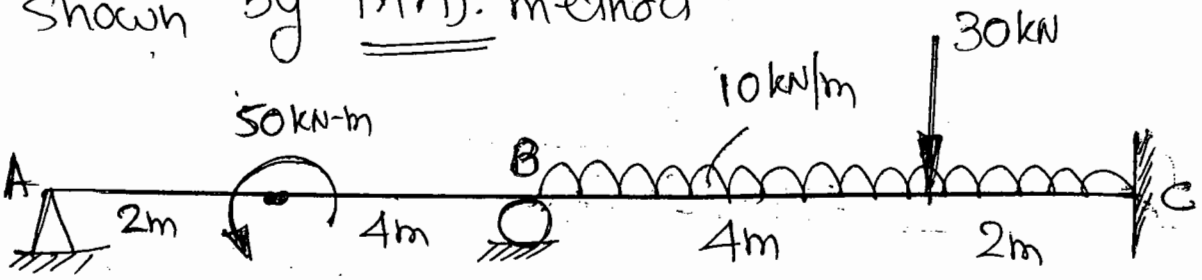
M.D. Table

A ✓ B ✓ ☆ { Simple Hinge C Roller }

AA'	AB	BA	BC	CB	Members
0	1	0.4	0.6		DF
50	-60	60	-86.25	108.75	FEM
			-54.37 ← -108.75		Release @ C.O
50	-60	60	-140.62	0	Initial Values
0	10 → 0	32.25 → 5	48.37 →	0	Bal C.O
	0 ←	-2 ←	-3 →	0	Bal C.O
50	-50	95.25	-95.25	0	Final Values

Refer S.D. Notes for SFD, BMD.

Eg:-4] Analyse the continuous beam shown by M.D. method.



Sol:2

(a) FEM

$$M_{FAB} = -\frac{Mb}{l^2}(2a-b) = \frac{-50\times4\,(2\times2-4)}{6^2} = 0$$

$$M_{FBA} = -\frac{Ma}{l^2}(2b-a) = -\frac{50\times2}{6^2}(2\times4-2) = -16.67 \text{ kN-m}$$

$$M_{FBC} = -\frac{wl^2}{12} - \frac{Wab^2}{l^2} = -43.33 \text{ kN-m}$$

$$M_{FCB} = +\frac{wl^2}{12} + \frac{Wa^2b}{l^2} = 56.67 \text{ kN-m}.$$

(b) D.F. (For Intermediate Support)

		k	Σk	$DF = \frac{k}{\Sigma k}$
B	BA	$\frac{3}{4}\left(\frac{I}{l}\right) = \frac{3}{4}\left(\frac{I}{6}\right) = 0.125I$	0.292I	0.43
	BC	$\left(\frac{I}{l}\right) = \frac{I}{6} = 0.167I$		0.57

M.D. Table

A (Hinge) ✓ B C (Fixed)

AB	BA	BC	CB	Members
-	0.43	0.57		DF
0	-16.67	-43.33	56.67	FEM
0 →	0			Release (A) C.O.
0	-16.67	-43.33	56.67	Initial Values
0 ←	25.80	34.20 →	17.10	Bal C.O
0	9.13	-9.13	73.77	Final Values

↓ ↓ ↓

Draw BMD, SFD.

Sinking and Rotation of Support

Additional Moment due to Rotation

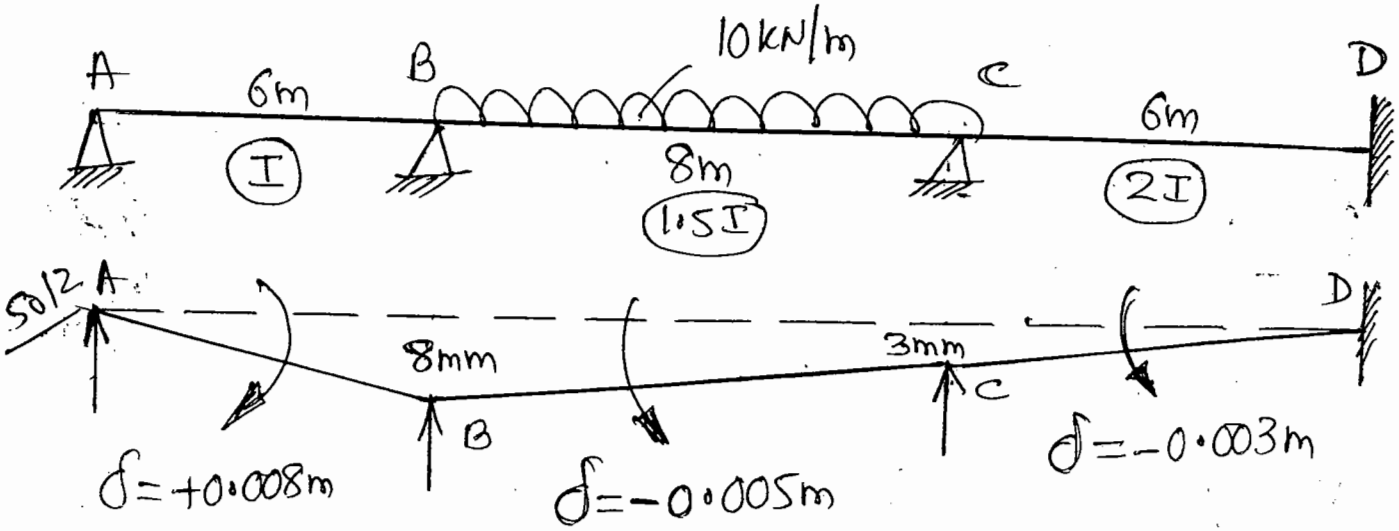
(i) Additional Moment at Near end $= \frac{4EI\theta}{l}$

(ii) Additional moment at Far end $= \frac{2EI\theta}{l}$

(iii) Additional moment due to Sinking $= -\frac{6EI\delta}{l^2}$

★ ∴ The above additional moments are added to F.E.M ★

Eg:- 5] Analyse the continuous beam shown by M.D. method and draw SFD, BMD. Support 'B' and 'C' settle by 8mm and 3mm respt. $EI = 2\times10^4\ kN/m^2$



(a) FEM

$$M_{FAB} = 0 - \frac{6EI\delta}{l^2} = 0 - \frac{6(1\times2\times10^4)(0.008)}{6^2} = -26.67$$

$$M_{FBA} = 0 - \frac{6EI\delta}{l^2} = 0 - \frac{6(1\times2\times10^4)(0.008)}{6^2} = -26.67$$

$$M_{FBC} = -\frac{wl^2}{12} - \frac{6EI\delta}{l^2} = \frac{-10\times8^2}{12} - \frac{6(1.5\times2\times10^4)(-0.005)}{8^2}$$

$$= -39.27\ kN\text{-}m$$

$$M_{FCB} = +\frac{wl^2}{12} - \frac{6EI\delta}{l^2} = \frac{10\times8^2}{12} - \frac{6(1.5\times2\times10^4)(-0.005)}{8^2}$$

$$= +67.40\ kN\text{-}m$$

$$M_{FCD} = 0 - \frac{6EI\delta}{l^2} = 0 - \frac{6(2\times10^7)(-0.003)}{6^2}$$

$$= +20 \text{ kN-m}$$

$$M_{FDC} = 0 - \frac{6EI\delta}{l^2} = +20 \text{ kN-m}$$

(b) D.F

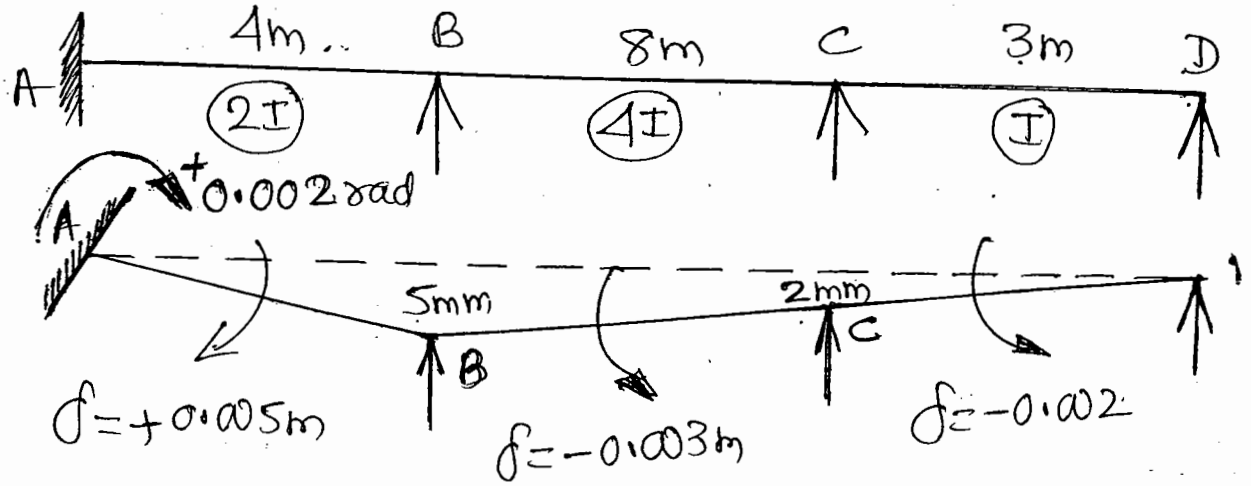
		k	Σk	DF = k/Σk
B	BA	$\frac{3}{4}\left(\frac{I}{l}\right) = \frac{3}{4}\times\frac{I}{6} = 0.125I$	0.3125I	0.4
	BC	$I/l = \frac{1.5I}{8} = 0.1875I$		0.6
C	CB	$I/l = \frac{1.5I}{8} = 0.1875I$	0.5000I	0.36
	CD	$I/l = \frac{2I}{6} = 0.333I$		0.64.

(c) M.D Table

AB	BA	BC	CB	CD	DC	Member
	0.4	0.6	0.36	0.64		DF
-26.67	-26.67	-39.27	67.40	20	20	FEM
+26.67 →	13.33					Release A, C.O
0	-13.33	-39.27	67.40	20	20	Initial
0 ←	21.04	31.56 → -31.46 ← -15.88	15.76	-55.94 →	-27.97	Bal, C.O
0 ←	6.35	9.53 → -5.67 ← -2.83	4.76	-10.08 →	-5.04	Bal, C.O
0 ←	1.13	1.70 → -1.71 ← -0.85	0.85	-3.04 →	-1.52	Bal, C.O
0 ←	0.34	0.51 → -0.30 ← -0.15	0.25	-0.54 →	-0.27	Bal, C.O
0	+0.06	0.09	-0.09	0.16		Bal
	15.59	-15.59	49.79	-49.48	-14.80	Final

Eg:- 6] fig shows a continuous beam ABCD. Analyse the beam by M.D method. If the End "A" rotates by 0.002 radians in the clockwise order & Support 'B' sinks by 5mm & 'C' by 2mm. Take

$EI = 18000\ kN\text{-}m^2$



(a) FEM

$$M_{FAB} = 0 + \underbrace{\frac{4EI\theta}{l}}_{\text{Near End}} - \underbrace{\frac{6EI\delta}{l^2}}_{\text{Sinking}}$$

$$= 0 + \frac{4(2\times18000)(0.002)}{4} - \frac{6(2\times18000)(0.005)}{4^2} = 4.5$$

$$M_{FBA} = 0 + \underbrace{\frac{2EI\theta}{l}}_{\text{Farend}} - \frac{6EI\delta}{l^2}$$

$$= 0 + \frac{2(2\times18000)(0.002)}{4} - \frac{6(2\times18000)(0.005)}{4^2} = -31.5$$

$$M_{FBC} = 0 - \frac{6EI\delta}{l^2} = 0 - \frac{6(4\times18000)(-0.003)}{8^2} = 20.25$$

$$M_{FCB} = 0 - \frac{6EI\delta}{l^2} = 20.25$$

$$M_{FCD} = 0 - \frac{6EI\delta}{l^2} = -\frac{6(1\times 18000)(-0.002)}{3^2} \quad (5)$$

$$= 24 \text{ kN-m}$$

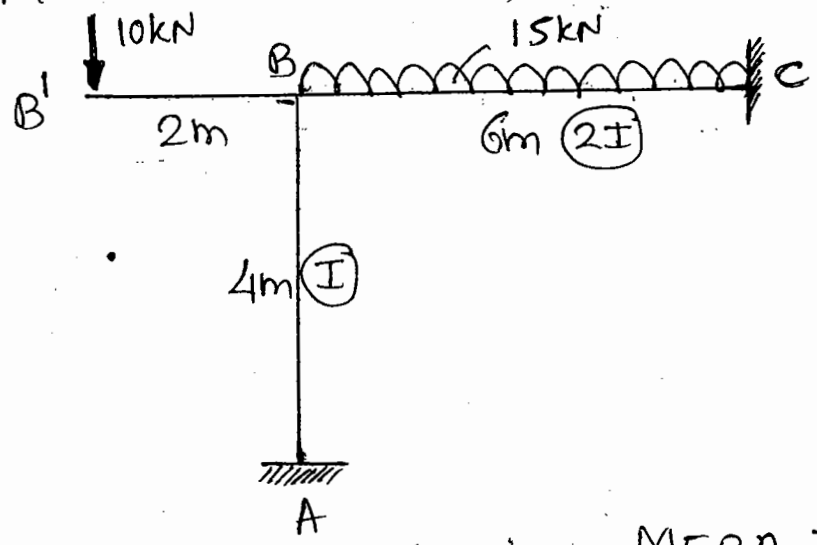
$$M_{FDC} = 0 - \frac{6EI\delta}{l^2} = 24 \text{ kN-m}$$

(b)

Date 05/10/18

: Non-Sway Frames :-

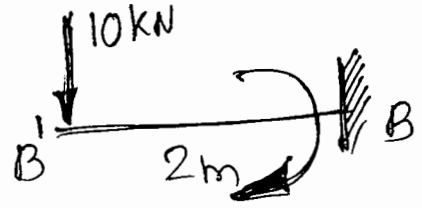
Eg:- 1] Analyse the rigid frame by M.D. method. Draw SFD, BMD & EC.



(a) FEM:

$$M_{FAB} = M_{FBA} = 0$$

$$M_{FBC} = \frac{-wl^2}{12} = -45, \quad M_{FCB} = +45 \text{ kN-m}$$



$$M_{BB'} = +10 \times 2 = +20 \text{ kN-m}$$

(clockwise resisting moment).

(b) D.F (For Intermediate)

		K	ΣK	$DF = \frac{K}{\Sigma K}$
B	BA	$I/l = I/4 = 0.25I$	0.58I	0.43
	BC	$I/l = \frac{2I}{6} = 0.33I$		0.57
	BB'	0		0

(c) M.D. Table

BB'	AB	BA	BC	CB	Member
0		0.43	0.57		DF
20	0	0	-45	45	FEM
0		-10.75	14.25		Bal
—	5.37	—	—	7.13	C.O
20	5.37	10.75	-30.75	52.13	Final Values.

At 'B'' $\boxed{M_{BA} + M_{BC} + M_{BB'} = 0}$

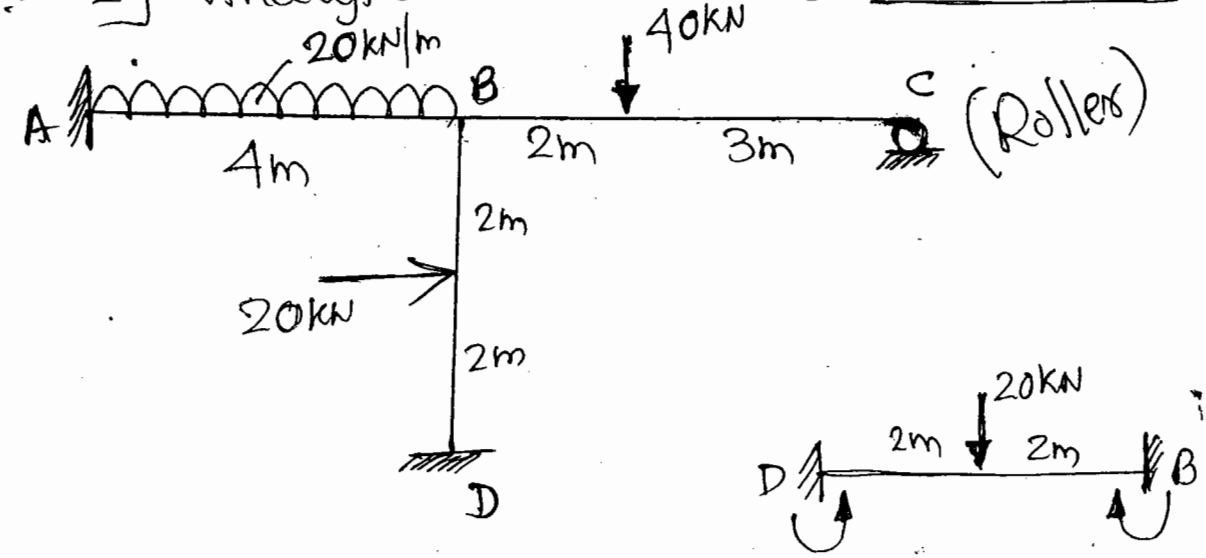
$0 - 45 + 20 = -25$

Refer S.D. method for Reaction,

SFD & BMD.

— x —

Eg:- 2] Analyse the frame by M.D. method



(a) FEM :

$$M_{FAB} = -\frac{wl^2}{12} = -26.67, \quad M_{FBA} = +26.67$$

$$M_{FBC} = -\frac{Wab^2}{l^2} = -28.8, \quad M_{FCB} = +\frac{Wa^2b}{l^2} = +19.2$$

$$M_{FDB} = -\frac{Wl}{8} = -10; \quad M_{FBD} = +10$$

(b) D.F : (For Intermediate)

		K	ΣK	$DF = \frac{K}{\Sigma K}$
B	BA	$I/l = I/4 = 0.25I$	$0.65I$	0.38
	BC	$\frac{3}{4}(I/l) = \frac{3}{4}(I/5) = 0.15I$		0.24
	BD	$I/l = I/4 = 0.25I$		0.38

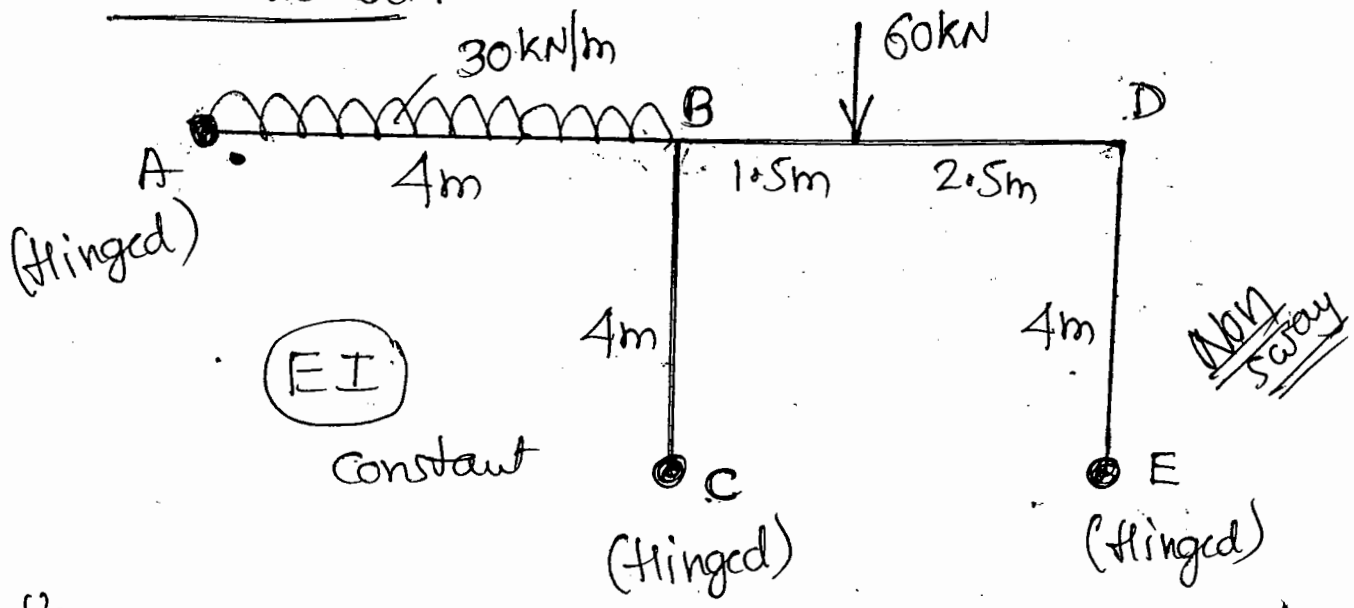
(c) "C" Roller

AB	BA	BD	DB	BC	CB	Member
	0.38	0.38		0.24		DF
-26.67	26.67	+10	-10	-28.8	19.2	FEM
					-19.2	Release @ C.O.
				-9.60		
-26.67	26.67	10	-10	-38.40	0	Initial Values.
0.33	0.66	0.66	0.33	0.41	0	Bal C.O
-26.33	27.33	10.66	-9.67	-37.99	0	Final Values.
					0	

Refer S.D. Notes For BMD.

At "B" $M_{BA} + M_{BC} + M_{BD} = 0$

Eg:- 3] Analyse the frame shown by MD method.



Sol:

(a) FEM

$$M_{FAB} = -\frac{wl^2}{12} = -40, \quad M_{FBA} = +40$$

$$M_{FBD} = -35.16, \quad M_{FDB} = +21.10 \text{ kN-m}$$

(b) D.F (For Intermediate)

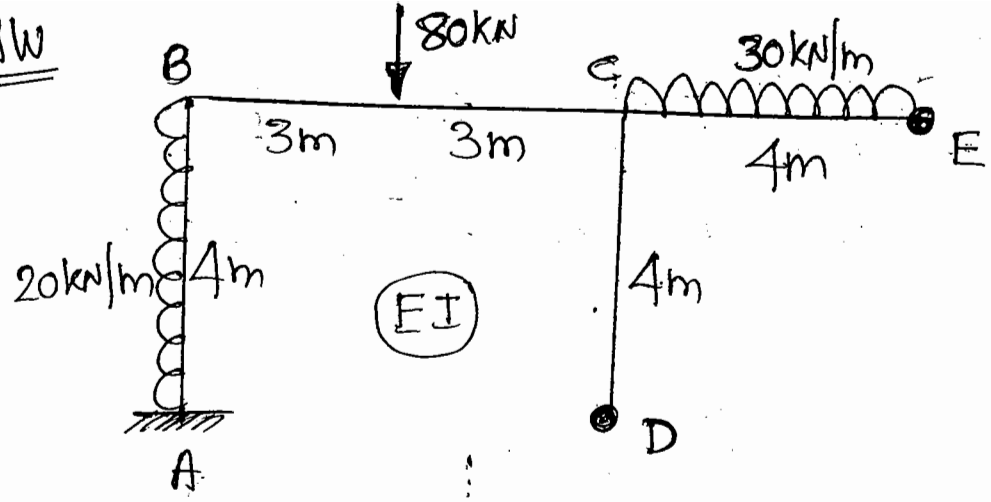
		K	ΣK	$DF = \frac{K}{\Sigma K}$
B	BA	$\frac{3}{4}(I/4) = 0.187I$	0.625I	0.30
	BC	$\frac{3}{4}(I/4) = 0.187I$		0.30
	BD	$I/4 = 0.25I$		0.40
D	DB	$I/4 = 0.25I$	0.437I	0.57
	DE	$\frac{3}{4}\left(\frac{I}{4}\right) = 0.187I$		0.43.

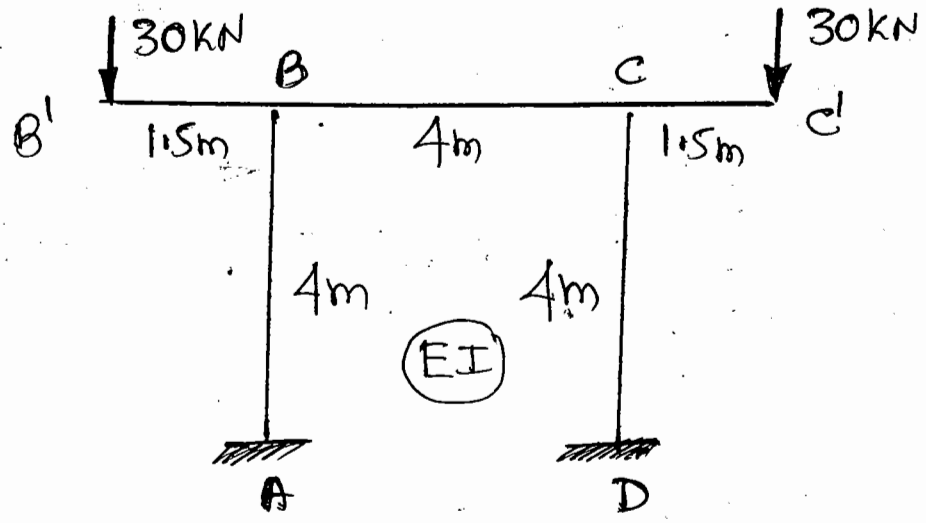
M.D.Table :

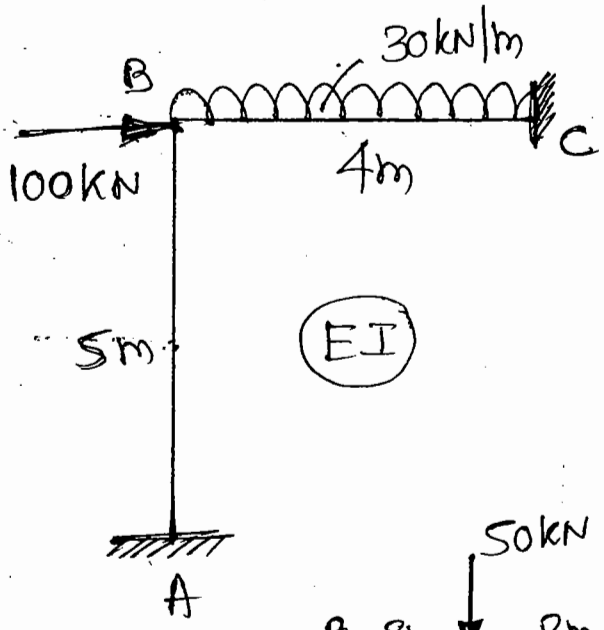
AB	BA	BC	CB	BD	DB	DE	ED	Member
	0.30	0.30		0.40	0.57	0.43		DF
-40	40	0	0	-35.16	21.10	0	0	FEM
+40 →	20							Release @ (A) C.O
0	60	0	0	-35.16	21.10	0	0	Initial
0 ←	-7.45	-7.45 →	0	-9.94 ↘↙ -6.01	-12.03 ↘↙ -4.97	-9.07 →	0	Bal C.O
0 ←	1.80	1.80 →	0	2.40 ↘↙ 1.42	2.83 ↘↙ 1.20	2.14 →	0	Bal C.O
0 ←	-0.43	-0.43 →	0	-0.57 ↘↙ -0.34	-0.68 ↘↙ -0.28	-0.52 →	0	Bal. C.O
0 ←	0.10	0.10 →	0	0.14 ↘↙ 0.08	0.16 ↘↙ 0.07	0.12 →	0	Bal C.O
	-0.02	-0.02		-0.04	-0.04	-0.03		Bal
0	54	-6	0	-48.02	7.36	-7.36	0	Final
	↻	↻		↻	↻	↻		

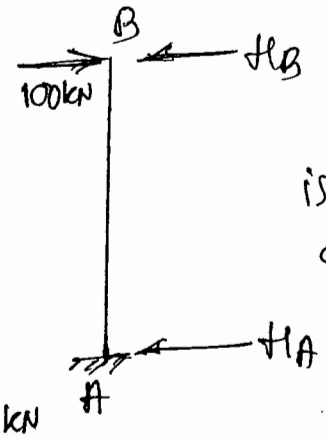
At "B" → $M_{BA} + M_{BC} + M_{BD} = 0$ | At "D" $M_{DB} + M_{DE} = 0$

HW









100 kN load is used only at the time of SFD.